

POVERTY ANALYSIS WITHIN A GENERAL EQUILIBRIUM FRAMEWORK

By

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Overview

The main objective of this paper is to show how Social accounting Matrixes and computable general equilibrium models can be used to highlight and address issues related to income distribution and poverty. The paper is divided into two major parts. The first part of the paper (sections 1B5) presents the concept of the social accounting matrix (SAM) as a comprehensive, consistent and disaggregated data system and shows how the SAM methodology can be used to analyse issues related to income distribution and, in a much more limited way, poverty. A prototype SAM reflecting the socioeconomic characteristics of an archetype African economy is postulated and used to illustrate the interrelationships among the structure of production, the factorial income distribution, the income distribution by socioeconomic household groups and the expenditure pattern of those groups.

The second part (sections 6B9) is devoted to the presentation of a computable general equilibrium (CGE) model calibrated on the archetype African SAM. One major innovation in the specification of the present CGE is that it goes part way in endogenizing the poverty line and the resulting poverty incidence among the different socioeconomic household groups. The model is used to simulate the impact of two exogenous shocks (a fall in the price of the export crop and an import tariff reform) specifically on poverty. An interesting feature of the model is that the effects of the shocks on such endogenous variables as product and factor prices; the structure and composition of output, exports and imports, and the pattern of employment are traced through in terms of their ultimate impact on poverty.

1. Social accounting, income distribution and poverty

The genesis of the Social Accounting Matrix (SAM) goes back to Richard Stone's pioneering work on social accounts. Subsequently Graham Pyatt and Erik Thorbecke (1976) further formalized the SAM and showed how it could be used as a conceptual and modular framework for policy and planning purposes.¹

The SAM is a comprehensive, disaggregated, consistent and complete data system that captures the interdependence that exists within a socioeconomic system. Depending on the classification scheme used to record transactions and the extent of disaggregation, the SAM can provide useful information about such key issues as intersectoral linkages (such as between agriculture and industry); inter-regional flows within an economy; the determination of the income distribution by socioeconomic groups given the structure and technology of production and the resource endowments of these groups; and the relationship between a given regional economy and other regional economies within a nation, and with the rest of the world.

Alternatively, the SAM can be used as a conceptual framework to explore the impact of

¹Section 2B5 of this paper draws heavily on Thorbecke (1995) and Thorbecke (1998).

exogenous changes in such variables as exports, certain categories of government expenditures, and investment on the whole interdependent socioeconomic system, e.g. the resulting structure of production, factorial and household income distributions. As such the SAM becomes the basis for simple multiplier analysis and the building and calibration of a variety of applied general equilibrium models. The SAM as a data system and as a conceptual framework is discussed in Section 2. Section 3 is devoted to a crucial issue in building and using a SAM, i.e., that of the appropriate classification and disaggregation scheme applying to the various accounts. The chosen taxonomy and the level of disaggregation depend critically on the questions that the SAM methodologies are expected to answer. If the SAM is to be used to explore issues related to income distribution then the household account is to be broken down into a number of relatively homogeneous household groups reflecting the socioeconomic characteristics of the country or region under consideration. Alternatively, if the purpose of the SAM is to analyse intersectoral linkages, then a relatively detailed sectoral disaggregation of production activities using such criteria as characteristics of the good or service produced and type of technology employed in production is called for. In Section 3, criteria relevant to building appropriate taxonomies for each of the major SAM accounts (i.e., production activities cum commodities, household, factors, government, capital, and rest of the world) are discussed.

Section 4 is devoted to a discussion of the different data sources needed to construct a SAM and the processes through which inconsistencies among these data sources (e.g., regional and national income accounts data, input-output information, household income and expenditure surveys, agricultural and industrial censuses) can be reconciled. The SAM is almost an ideal instrument within which consistency checks among different data sources can be undertaken, inconsistencies reconciled and data gaps identified. Often these data gaps can be remedied through new surveys and other types of data collection and errors corrected particularly when the preparation of the SAM is institutionalized within a Central Statistical Bureau, as is presently the case in Indonesia.

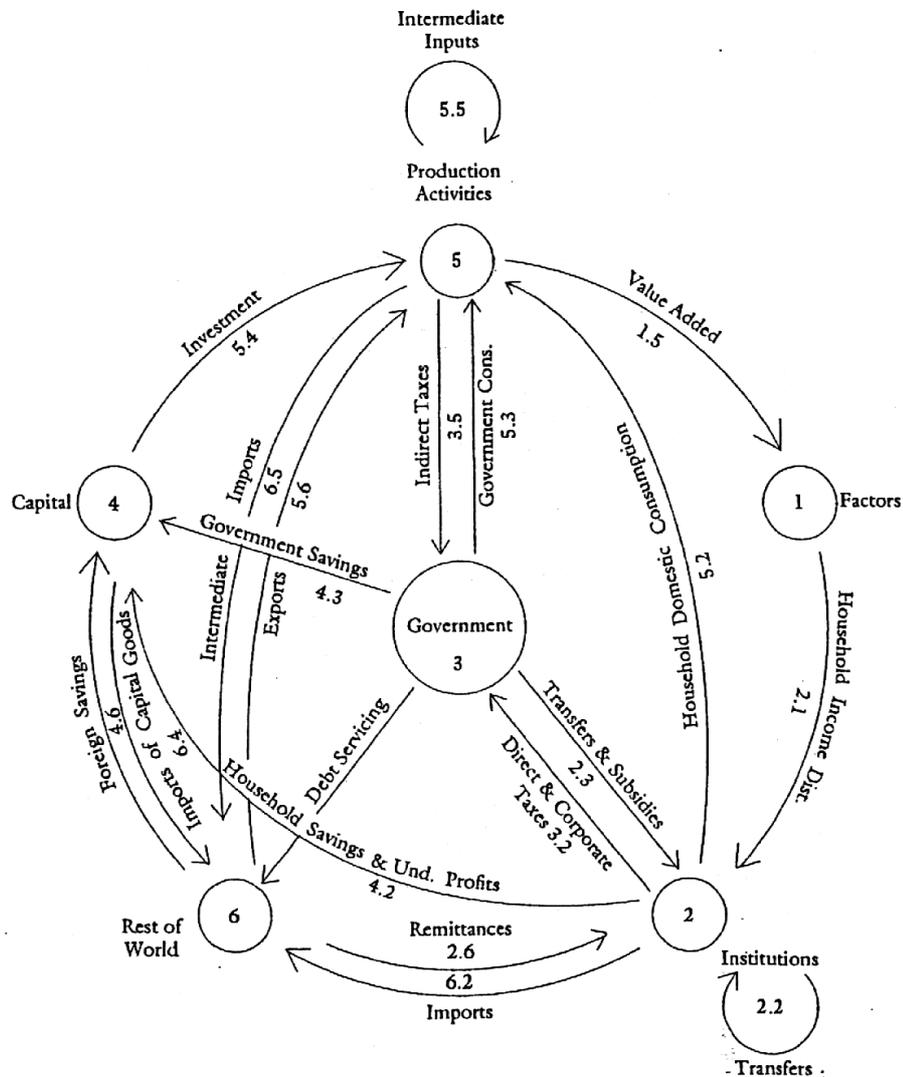
In Section 5 the SAM-based multiplier methodology is presented. In particular the impact of exogenous shocks such as exports, government programmes and investment on the structure of production, the factorial and the income distributions is analysed. A SAM of an archetype African economy is presented and used to explore the multiplier impact of different shocks. This archetype SAM provides the foundations upon which the CGE model is calibrated, beginning in Section 6. Section 7 discusses the income distribution and poverty measures used in the paper. Section 8 is devoted to the presentation of the general equilibrium model, and Section 9 analyses the impact on poverty of two different simulated shocks, a fall in the world price of export and a trade reform affecting an archetype African economy. Section 10 is devoted to some conclusions.

2. The overall conceptual framework

As a data framework, the SAM is a comprehensive and disaggregated snapshot of the socioeconomic system during a given year. It provides a classification and organizational scheme for the data useful to analysts and policy makers alike. It explicitly incorporates various crucial relationships among variables such as the mapping of the factorial income distribution from the structure of production and the mapping of the household income distribution from the factorial income distribution. Table 1 presents a basic

SAM. It can readily be seen that it incorporates all major transactions within a socioeconomic system. Whereas the SAM in Table 1 is a snapshot of the economy, Figure 1, which reproduces all of the transformations appearing in Table 1, can be interpreted more broadly as representing flows (over time) that, in turn, have to be explained by structural or behavioural relationships.

Figure 1
Flow Diagram of SAM Transactions



The flow diagram reflects exactly the transactions and transformations appearing in the SAM on Table 1. Note that transactions are numbered in a way consistent with the numbering of the Accounts in Table 1. For example, the allocation of value added is a receipt for the Factor Account (#1) and a payment by the Production Activities Account (#5); hence, the corresponding transformation (matrix) is denoted by 1.5.

Source: Thorbecke (1988)

Table 1: A basic social accounting matrix (SAM)

				Expenditures						Totals		
				1	2a	2b	3	4	5		6	
				Factors of production	Institutions						Production activities	Rest of the world combined account
					Current accounts				Combined capital accounts			
Households	Companies	Governments										
Receipts	1	Factors of production							Value added payments to factors	Net factor income received from abroad	Incomes of the domestic factors of production	
	2a	Institutions	Current accounts	Households	Allocation of labour income to household	Current transfers between households	Profits distributed to domestic households	Current transfers to domestic households			Net non-factor incomes received from abroad	Incomes of the domestic institutions after transfers
	2b			Companies	Allocation of operating surplus to companies			Current transfers to domestic companies				
	3			Government		Direct taxes on income and indirect taxes on current expenditures	Direct taxes on companies plus operating surplus of state enterprises		Indirect taxes on capital goods	Indirect taxes on inputs	Net non-factor incomes received plus indirect taxes on exports	
	4			Combined capital account		Household savings	Undistributed profits after tax	Gov't current account surplus			Net capital rec'd from abroad	
	5	Production activities			Household consumption expend. On dom. goods		Government current expenditure	Investment expenditures on domestic goods	Raw material purchases of domestic goods	Exports	Aggregate demand – gross outputs	
6	Rest of the world combined account			Household consumption expend. On imp. goods			Imports of capital goods	Imports of raw materials		Imports		
Totals				Incomes of the domestic factors of production	Total outlay of households	Total outlay of companies	Total outlay of government	Aggregate Investment	Total costs	Total foreign exchange receipts		

Source: Thorbecke (1988).

Table 1 presents all of these flows in a basic SAM. A SAM is a square matrix in which each transactor or account has its own row and column. The payments (expenditures) are listed in columns and the receipts are recorded in rows. As the sum of all expenditures by a given account (or subaccount) must equal the total sum of receipts or income for the corresponding account, row sums must equal the column sums of the corresponding account. For example, the total income of a given institution (say a specific socioeconomic household group) must equal exactly the total expenditures of that same institution. This is the economic analog of the physicists' law of conservation of energy. Hence, analysts interested in understanding how the structure of production influences the income distribution can obtain useful insights by studying the SAM.

In the basic SAM of Table 1, six accounts are distinguished. Production activities produce different sectoral goods and services (e.g., textile products) by buying raw materials and intermediate goods and services (from the region under consideration, other regions within the nation and from abroad). In addition these accounts pay indirect taxes to the government and the remainder is, by definition, value added that is distributed to the factors of production (see column 5). Production activities receipts (row 5) derive from sales to households, exports and the government. In the present formulation of the SAM no distinction is made between production activities and commodities. For the sake of simplicity, it is assumed that a production activity is equivalent to a corresponding commodity. In some instances, the SAM format distinguishes between production activities and commodity accounts. This would be the case when a given production activity produced different commodities, for example, so that these two sets of accounts would require different sectoral breakdowns. For this reason, many SAMs include both production activities and commodities accounts. When commodity accounts appear in a SAM they can best be seen as representing a region's or nation's product markets. Thus the SAM of the archetype African economy presented here includes both a production and commodity accounts.

Factors of production accounts typically include labour and capital subaccounts. They receive income (recorded in row 1) from the sale of their services to production activities in the form of wages, rent and net factor income received from abroad or from other regions (corresponding to the value added generated by the production activities). In turn, these revenues are distributed (col. 1) to households as labour incomes and to companies as distributed profits.

Institutions include households (typically further broken down by socioeconomic groups), companies (i.e., firms) and the government. From row 2a, it can be seen that households receive factor income (wages and other labour income, rent, interest and profits) as well as transfers from government and from the rest of the nation and world (e.g., remittances). Households' expenditures (in column 2a) consist of consumption of goods from the region, from other regions and from abroad, and income taxes with residual savings transferred to the capital account. Companies (2b) receive profits and transfers and spend on taxes and transfers with their residual savings channeled into their capital account.

The government account (3) is distinct from administrative public activities included in the production activities' account. These public services (such as education) buy intermediate goods, pay

wages, and deliver public and administrative services. The government account per se allocates its current expenditures to buying the services provided by the production activities account. Other government expenditures (col. 3) are transfers and subsidies to households and companies and the remaining savings are transferred to the capital account. On the income side, the government receives tax revenues from a variety of sources and current transfers from abroad (row 3).

The fifth account is the combined capital account. On the income side (row 4) it collects savings from households, companies and the government, as well as foreign and other regions' savings and, in turn, channels these aggregate savings into investment (col. 4).

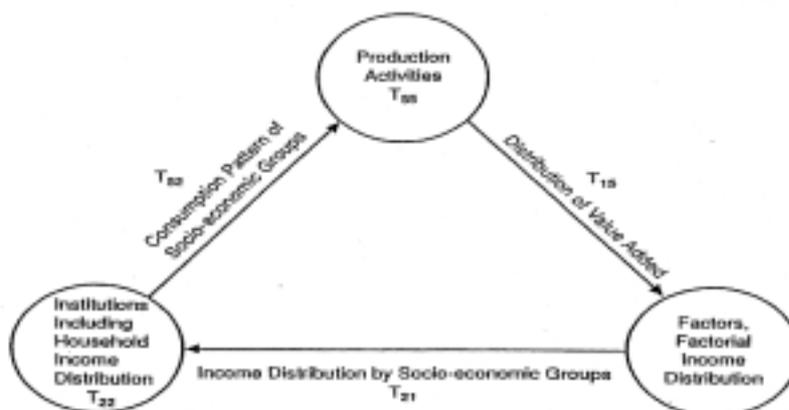
Finally, transactions between domestic residents and foreign residents, respectively, are recorded in the rest of the world accounts (6). These transactions include, on the receipt side, households' consumption expenditures on imported final goods as well as imports of capital goods and raw materials (row 6). The economy receives income from the rest of the nation and world (col. 6) from exports and factor and nonfactor income earned. The difference between total foreign exchange receipts and imports is by definition net capital received from abroad or the rest of the nation and extra-regional and foreign savings.

The SAM framework can also be used as a conceptual framework and as a basis for modeling. In this case the generating mechanisms influencing the flows appearing in Figure 1 have to be spelled out explicitly and quantitatively.

The first question to address in a SAM-based framework is which accounts should be considered exogenous and which endogenous. It has been customary, and it is certainly logical in the context of this specific study, to consider the government, the rest of the world and the capital account as exogenous and the factors, institutions and production activities' accounts as endogenous. To illustrate how the SAM approach lends itself to deriving the ultimate income distribution and expenditure pattern by socioeconomic groups following, say, a change in the structure of production resulting from government actions or a change in exports, distinguishing between the determination of primary and secondary income distribution is useful. Thus, a distinction is drawn between *primary* claims on resources that arise directly out of the productive process of work and accumulation, and *secondary* claims that result from the transfer of primary claims. The former results from prevailing patterns of (1) production and (2) resource endowment (human capital, physical capital and land) among households.

The primary income distribution is determined through the triangular interrelationship linking production activities, factors and households. In Figure 1 this interrelationship appears as the value added flow (denoted by arrow 1.5) from production activities to factor incomes; from the latter to household income determination and distribution (2.1), which yields, ultimately, the household domestic consumption pattern (5.2). While the primary income distribution is by far the most important determinant of incomes received by the various socioeconomic groups, a secondary income distribution may work through the family, village, or, more important, through the state in the form of transfers and subsidies (2.3) and taxes (3.2). Figure 2 reproduces this same key *triangular*

Figure 2 Simplified relationship among Principal SAM accounts (Productive activities, Factors and Institutions)^a



Source: Thorbecke (1992).

interrelationship among production activities, the factorial income distribution and that household income distribution that is emphasized throughout this paper.

If we are to understand and explain, in an operational way, the mechanisms through which these transformations occur, great care must be exercised in designing appropriate classification schemes for each of the three endogenous accounts. These transformations incorporate the mechanisms that translate the generation of value added by production into the incomes of different types of households and other institutions. The link is provided by factors of production. The level and structure of output by the different activities generate the aggregate demand for labour of different types, natural resources and capital services. Hence, employment enters into the analysis. The stream of value added, from the production side, rewards the factors of production, with wages going to different types of labour, rent going to land and other resources, and profits to capital. In this way a picture is obtained of the

factorial distribution of income, which is captured in Table 1 by the interface between column 5 and row 1 and, analogously, by matrix T_{15} in Figure 2. With regard to production activities, four criteria suggest themselves in deriving an appropriate classification: (1) the nature of the item produced be it a good, service or commodity; (2) the type of technology used, in terms of labour and capital intensity; (3) the form of organization underlying the production process (i.e., farm or firm relying on family labour and self-employment, as opposed to an incorporated or even a state enterprise); and (4) whether the commodities are tradeable or non-tradeable.

In turn, the classification of factors and households should be consistent with our interest in employment and equity issues as they relate more particularly to rural areas—as poverty is endemic in these areas. With the qualification that any ultimate taxonomy should be country specific, the following breakdown of factors may be suggested: (1) family labour (further broken down between unpaid and paid and self-employed and hired, and, if possible, distinguishing between male and female labour); (2) unskilled labour (with some of the same additional distinctions as in the category above); (3) skilled labour; and (4) capital (which could be land or other forms of capital).

Translation from factorial distribution to the distribution of incomes across institutions, and particularly across different household groups, depends on which institutions own which factors. Thus, for example, wage payments to unskilled labour go to the households that provide semi-skilled labour; imputed labour income is received by small farmers from the services performed by self-employed family labour on their own farms. Rent income (whether imputed or not) accrues to the owners of land and other natural resources, and finally, profits accrue to owners of capital. This second transformation is shown in Table 1 by the interface between column 1 and row 2, as well as by matrix T_{21} in Figure 2. Three main criteria appear important in classifying households: location; resource endowment and wealth; and occupation of the head of the household. Location, particularly between rural and urban areas, is a crucial criterion largely on the grounds that policy often has a locational element and often an urban bias. Resource endowment is important at several levels. Access to land is a critical consideration in rural areas and the landless can be affected quite differently from the smallholder, or large farmers, by development policy. Likewise, the better educated in both urban and rural areas are able to land jobs in formal and organized activities, whereas the uneducated are largely limited to employment opportunities in traditional agriculture and informal urban activities. The endowment of land and human capital is a crucial determinant of the ultimate income distribution and standards of living of the various socioeconomic household groups.

A third transformation in Figure 2 yields the consumption pattern of the different socioeconomic groups (interface between column 2a and row 5 in Table 1 and matrix T_{52} in Figure 2). It reveals the value of the commodities (assumed here to be equivalent to production activities) consumed by these groups. This transformation provides crucial information on the living standards of the various groups and the extent to which they are able to satisfy their basic needs. Thus, in the CGE model presented in Part 2 we specify consumption functions of the linear expenditure type that predetermine a basic needs income. Two final endogenous transformations appear in Figure 2 reflecting transfers occurring within, respectively, the production activities account and the institutions account. T_{55} represents the matrix of intermediate demand by production activities and is nothing else

than the conventional input-output table. T_{22} captures transfers among institutions and, in particular, transfers from some relatively better off socioeconomic groups to other poorer groups.

At this stage, one qualification needs to be made. Whereas the SAM approach explains the determination of total incomes accruing to the various socioeconomic groups, it does not generate the intra-group income distributions. To the extent that poverty tends to be concentrated in a few groups, such as the landless and small farmers in rural areas and the informal sector workers in urban areas, between-group variance is likely to explain a reasonably high proportion of total income variance in society. If one wants to approximate more exactly the impact on poverty of measures affecting the structure of production, knowledge of the income distributions within socioeconomic groups is necessary because poor households (those with incomes below a given normative poverty line) are likely to be found even in socioeconomic groups enjoying average income levels significantly higher than the poverty line. Thus, in Section 8, where a model of an archetype African economy is specified, we postulate the intra-household group income distributions for the different socioeconomic groups.

3. Classification and disaggregation of accounts

Classification matters in a fundamental sense whether the SAM is used as a diagnostic tool to understand better the underlying interdependent socioeconomic structure of an economy, or as a conceptual framework and basis for modeling. Economic concepts and variables must be represented in a SAM by appropriately corresponding classes and categories. To each conceptual framework, there must be a corresponding taxonomic and data system.

What are some of key issues in deciding on a SAM classification scheme? First, the level and extent of disaggregation deserve consideration. In many instances, given the policy issues a SAM is supposed to address, fairly aggregative SAMs broken down in relatively few categories will do. However, since it is always possible to consolidate and aggregate subaccounts—but not the other way around—it may be better to start at a level of disaggregation that is as detailed as data reliability allows. Secondly, the degree of homogeneity is crucial in the design of classifications. For example, in a classification of household groups, one would like to identify groups that are relatively homogeneous in terms of income sources and levels and expenditure patterns.

It has been argued that every classification should meet certain requirements if it is to be used in a SAM. A SAM taxonomy should (1) correctly reproduce the socioeconomic and structural (production) stratification within the society and economy; (2) distinguish relatively homogenous groups and categories; (3) be composed of socioeconomic groups that are recognizable for policy purposes and useful for socioeconomic analysis (i.e., specific target groups should be identified); (4) be based on comparatively stable characteristics that can be measured relatively easily and reliably; and (5) be derived from (a combination of) existing data sources (Alarcon et al., 1986).

Applying these criteria to household groups, it is noteworthy that a household classification based on income or expenditure brackets does not satisfy any of these requirements, except perhaps the first one. Since the poorest segment of society (say the bottom decile of the income pyramid) may

include very different household heads such as a landless agricultural worker or an urban informal sector worker, policies aimed at improving conditions in the two cases are likely to be very different.

There is no unique (standard) classification scheme or way of disaggregating and organizing the data in a SAM. The taxonomy used in any given SAM depends on the prevailing country or region specific characteristics and the objectives of the studies underlying the building of the SAM. In a SAM that emphasizes intersectoral linkages, the level of disaggregation of production activities needed to capture the structure of production is likely to be much smaller in poor developing countries than in an industrialized one. A SAM that is supposed to be used as a basis for exploring income distribution issues needs a finer disaggregation of socioeconomic household groups than one not highlighting income distribution.

Each account appearing in Table 1 can be disaggregated. A common approach is to start with selecting the most robust and appropriate classification criteria and then breaking them down into subcriteria and sub-subcriteria following a hierarchical top-down tree structure. The major criteria and subcriteria typically used in the classification and disaggregation of the different accounts include:

- *Production activities cum commodities*
 - Production activities: Two-digit or three-digit International Standard Industrial Classifications (ISIC), further broken down according to technology level (e.g., distinguishing between formal and informal technologies for the same type of product; size of firms in terms of number of employees; domestic vs. foreign owned; location; tradeable vs. non-tradeable).
 - Commodities: Nature of the good or service fulfilling similar needs; tradeable vs. non-tradeable; local vs. imported.
- *Institutions*
 - Households: Location (e.g., rural vs. urban); asset ownership (particularly land and ownership in the rural areas and human capital in urban areas); characteristics of the head or main earner, distinguished by main employment status, main occupation, main branch of industry and educational attainment, sex, main language, race (tribal) kinship. (See Figure 3.)
 - Companies: Ownership (distinguishing between national and foreign, and private and public status, respectively); legal status (incorporated vs. unincorporated and some family enterprises).
 - Government: Central vs. local and breakdown by capital and current expenditure categories.
- *Factors of production*
 - Labour: Occupation (distinguished by skill level and occupational category); wage

- employment vs. self-employment; location; education; sex; age; type and size of firm.
- Land and other natural resources: Land type and fertility; size of the holding; location.
- Capital: Domestic vs. foreign, private vs. public, and type or vintage of capital good.

Figure 3: Household characteristics

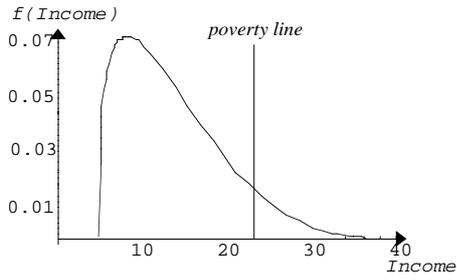


Figure 3a : Income distribution rural households

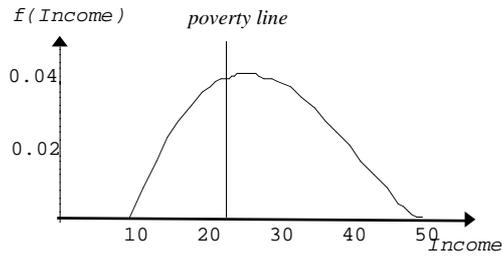


Figure 3b : Income distribution small landowner households

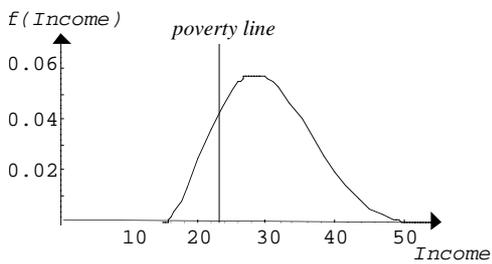


Figure 3c : Income distribution large landowner households

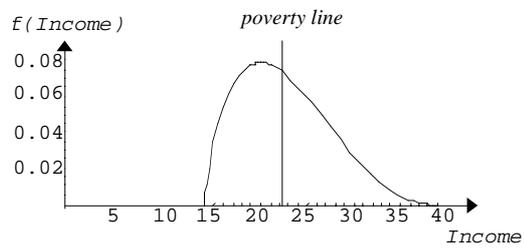


Figure 3d : Income distribution urban low education households

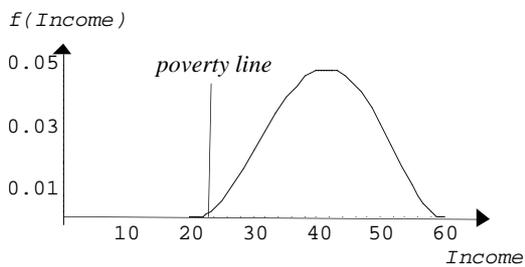


Figure 3e : Income distribution urban high education households

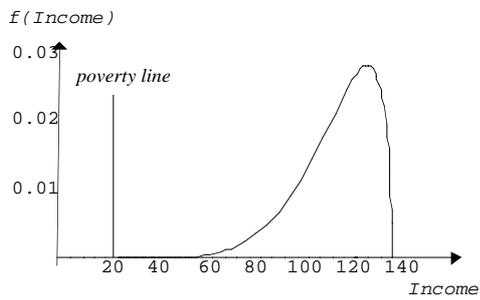


Figure 3f : Income distribution capitalist households

A great strength of the SAM is that it explicitly breaks down households into relatively homogenous socioeconomic categories that are recognizable for policy purposes and exhibit relatively stable characteristics. This type of disaggregation allows the SAM to be used to analyse the effects of government policies on income distribution. Although any classification is essentially arbitrary, there are many instances of effective classification such as the Standard Industrial Classification and the Standard Occupational Classification designed and used by Central Statistical Bureau. Recently the community of statisticians designed and recommended the adoption of a hierarchical classification of households that shows a top-down tree structure at different levels. This proposed taxonomy is reproduced in Figure 3. (For an interesting discussion of the importance of an appropriate household taxonomies, see Duchin, 1996.)

A final key issue that goes to the heart of defining and deciding on the domain of the SAM and that transcends accounts is that of regionalization.² While most SAM studies have been undertaken with national objectives in mind, it has been realized that distinguishing regions within a country SAM can enhance both its realism and its usefulness. If the economy displays significant regional differences in the types of goods produced, structure of production and technology, these differences could affect the standards of living of different household groups. Another important advantage of the explicit inclusion of the regional dimension in a SAM conceptual framework is that a large number of policy means tend to be location-specific. These may include investment projects, current government expenditures on services such as health and education, and price policies with respect to commodities and inputs at least to the extent that the production of specific commodities is regionally concentrated.

4. Data requirements in the construction of a SAM

A variety of data sources are required to build a SAM. Because the methods used in collecting and generating statistics differ significantly from one source to another (such as national income accounts, input-output, census information, surveys, etc.) the process of building a SAM provides a natural check on the mutual consistency of these sources and identifies possible data gaps and errors. In this sense the process of reconciliation that is endemic in generating a SAM has social value in its own right.³

There are different techniques for reconciling and forcing consistency within a SAM that does not balance—the most naïve and mechanical one being the RAS technique. Generally, it is far preferable to use judgements rather than mechanical approaches in ensuring that a SAM is consistent and balanced.

²This section on regionalization draws on Thorbecke (1985).

³In this connection, it is relevant to note that when a team of resident experts attached to the CBS in Jakarta was trying to build the first SAM for Indonesia in the late 1970s, the local Indonesian statisticians only became interested in and supportive of the exercise when they realized that the SAM provided an ideal framework within which to check data consistency and help reconcile inconsistencies. Soon thereafter the process of building SAMs was institutionalized within the CBS and so far four large-scale, highly disaggregated SAMs have been prepared and published by the CBS (for 1975, 1980, 1985 and 1990, respectively).

Given the degree of country or regional specificity and the numerous different objectives that construction of the SAM may have, it is not possible to identify a unique and general set of required data. The more disaggregated a SAM is intended to be, the more extensive are the data requirements. Some scholars maintain that "In all cases, the starting point should be the building of a highly aggregated SAM based on the country's national accounts statistics" (Sadoulet and de Janvry, 1995: 280) Others would contend that a more accurate and sensible approach for regional and inter-regional analyses and even national analysis is to construct a SAM region by region with inter-regional flows increasingly disaggregated.

There is no optimal sequence in which to proceed with the construction of a SAM. A good starting point is with the production activities account since the SAM can be seen as a major expansion on and extension of an IBO matrix. This would be particularly true when building a SAM for a region as opposed to a country. A recent IBO table can provide the basis for matrix T_{55} appearing in the basic SAM in Table 1 above. In particular, the IBO table will provide the needed information to fill in the appropriate production activities row sums in representing the vector of aggregate demands and the corresponding vector of column sums yielding the vector of aggregate supplies (sectoral outputs).

A second step might consist of breaking down value added (matrix T_{15} in Table 1) into income accruing to different labour categories and profits and rent going to one or more capital categories with the help of employment surveys and agricultural and industrial syntheses.

A third step could yield the incomes of the various socioeconomic groups relying on household income and expenditure surveys. Particularly crucial, in this context, is the mapping of the household income distribution from the factorial income distribution (T_{21}). On the household expenditure side, again consumption surveys together with information on taxes available from the government budget should provide the main spring for filling out column 2a of Table 1. For companies, most SAMs aggregate all firms into one category and the information needed to fill in column and row 2b in Table 1 is normally available from national accounts data. The government budget and additional public finance information relating to the sources of government revenues and the composition of government expenditure should yield the required figures for the government account (row 3 and col. 3). Finally, a detailed balance of payments supplemented by disaggregated trade statistics should make it possible to record transactions with the rest of the world.

When all the cells (submatrixes) of the SAM are filled in based on the primary information for all accounts except for one account (say the capital account), the income row and expenditure column of this last account appear as by magic (a conceptual requirement under Walras' Law). The recorded entries in the SAM for the capital account can then be checked against whatever primary information is available relating to any specific receipt or outlay of that account.

A final data and formatting issue is that the great majority of the existing SAMs contain only a rudimentary breakdown of financial transactions. When one of the objectives of the SAM is to highlight the flow of funds among various financial institutions, households and firms and the portfolios of different financial assets of these institutions, a financial SAM needs to be built.

5. SAM multiplier analysis and application to an African archetype economy

If a certain number of conditions are met—in particular, the existence of excess capacity and unemployed or underemployed labour resources—the SAM framework can be used to estimate the effects of exogenous changes and injections, such as an increase in the demand for a given production activity, government expenditures or exports on the whole system. As long as excess capacity and a labour slack prevail, any exogenous change in demand can be satisfied through a corresponding increase in output without having any effect on prices. Thus, for any given injection anywhere in the SAM, influence (e.g., an increase in the export demand for textile products, a government investment or private project leading to an increase in the production of food crops, or a subsidy or transfer accruing to a specific socioeconomic household group) is transmitted through the interdependent SAM system.

The total direct and indirect effects of the injection on the endogenous accounts, i.e., the total outputs of the different production activities and the incomes of the various factors and socioeconomic groups, are estimated through the multiplier process. For example, a public works programme resulting in the construction of a new rural farm-to-market road would require, among others, a significant amount of unskilled labour, which is typically provided by the landless and small farmers' household categories. In turn, a significant part of the incremental incomes earned by these two socioeconomic groups from their work on the road project is spent on food demand. The subsequent increase in good production to satisfy that demand leads to still further employment and income increments for these groups, and so on, until the multiplier process dampens.

To derive and illustrate the underlying logic of this methodology, let us at the outset assume, following the previous discussion in Section 2, that the only three accounts that are endogenously determined are production activities, factors and institutions (households and companies), while all other accounts are exogenous (government, capital, and the rest of the world). The resulting simplified SAM is presented in Table 2. Thus the simplified and truncated SAM consolidates all exogenous transactions and corresponding leakages and focuses exclusively on the endogenous transactions and transformations. Five endogenous transformations appear in Table 2. Note that the three exogenous accounts have been combined in Table 2 and the sum of the exogenous injections from government expenditures, investment and exports, respectively, has been consolidated into three vectors, x_1 , x_2 and x_3 . The first vector (x_1) represents the total exogenous demand for factors (and hence income injection to reward factors). Similarly, x_2 and x_3 represent, respectively, the total exogenous income accruing to the different socioeconomic household groups and companies from, say, government subsidies and remittances from abroad, and the total exogenous demand for the production activities (commodities) resulting from government consumption, investment and export demand. Likewise l'_i represents the corresponding leakages, from savings, imports and taxation.

Table 2: Simplified schematic social accounting matrix

				Expenditures				
				Endogenous accounts			Exog.	Totals
				Factors	Institutions	Production activities	Sum of other accounts	
					Households and companies			
1	2	3	4	5				
Receipts	Endogenous accounts	Factors	1	0	0	T_{13}	x_1	y_1
		Institutions, i.e., households and companies	2	T_{21}	T_{22}	0	x_2	y_1
		Production activities	3	0	T_{32}	T_{33}	x_3	y_3
	Exog	Sum of other accounts	4	1_1	$1_{1/2}$	$1_{1/3}$	1	Y_1
	Totals	5	y_1	$y_{1/2}$	$Y_{1/3}$	Y_1		

Source: Thorbecke (1995).

The logic underlying the scheme in Table 2, as will be seen shortly, is that exogenous changes (the x_i 's) in Table 2 determine, through their interaction within the SAM matrix, the income of the endogenous accounts, i.e., the production activities (vector y_3); the factor incomes (y_1); and the household and companies incomes (y_2).

For analytical purposes, the endogenous part of the transaction matrix is converted into the corresponding matrix of average expenditure propensities or coefficients. These can be simply obtained by dividing a particular element in any of the endogenous accounts by the total income for the column account in which the element occurs. From Table 2 it can be seen that A_n is partitioned as follows (i.e., A_n is composed of different subsets of coefficients):

$$A_n = \begin{bmatrix} 0 & 0 & A_{13} \\ A_{21} & A_{22} & 0 \\ 0 & A_{32} & A_{33} \end{bmatrix} \quad (1)$$

The subset A_{33} is the set of input-output coefficients reflecting the cents worth of inputs per dollar of each production activity's output. The subset A_{13} is the set of cents worth of primary inputs per dollar of output of each production activity. The coefficients of the subset A_{32} show, on average, the cents worth of each commodity (production activity) that each (socioeconomic) household group purchases with each of its dollars of total expenditures. The coefficients of the subset A_{22} shows, on average, the cents worth of income transfers to other household groups per dollar of income. Finally, A_{21} shows the cents worth of each dollar earned by each type of resource (primary input) that is allocated to each of the household groups.

From the definition of A_n , it follows that in the transaction matrix, each endogenous total income (y_n) is given as:

$$y_n = A_n y_n + x \quad (2)$$

which states that row sums of the endogenous accounts can be obtained by multiplying the average expenditure propensities for each row by the corresponding column sum and adding exogenous income x .

Equation 2 can be rewritten as

$$\begin{aligned} y_n &= (1 - A_n)^{-1} x \\ &= M_a x \end{aligned} \quad (3)$$

Thus, from Equation 3, endogenous incomes y_n (i.e., production activity incomes, y_3 , factor incomes, y_1 , and institution incomes, y_2 , as shown in Table 2) can be derived by premultiplying injection x by a multiplier matrix M_a . This matrix has been referred to as the *accounting multiplier matrix* because it explains the results obtained in a SAM and not the process by which they are generated. The latter would require the specification of a dynamic model including the different SAM accounts and variables.

One limitation of the accounting multiplier matrix M_a as derived in Equation 3, is that it implies unitary expenditure elasticities (the prevailing average expenditure propensities in A_n are assumed to apply to any incremental injection). While this assumption may be defensible for all other elements of

A_n , it is certainly unrealistic for the expenditure pattern of the household groups (A_{32}). A more realistic alternative is to specify a matrix of marginal expenditure propensities (C_n below) corresponding to the observed income and expenditure elasticities of the different agents, under the assumption that prices remain fixed.⁴ In this case, C_n formally differs from A_n in the following way: $C_{13} = A_{13}$, $C_{33} = A_{33}$, $C_{22} = A_{22}$, $C_{21} = A_{21}$, but $C_{32} \neq A_{32}$.

Expressing the changes in incomes (dy_n) resulting from changes in injections (dx), one obtains

$$\begin{aligned} dy_n &= C_n dy_n + dx \\ &= (I - C_n)^{-1} dx = M_c dx \end{aligned} \quad (4)$$

M_c has been coined a *fixed price multiplier matrix* and its advantage is that it allows any non-negative income and expenditure elasticities to be reflected in M_c .

Table 3 presents an illustrative example of a SAM for an archetype African developing economy. Although it was calibrated to reflect approximately the socioeconomic structure of Côte d'Ivoire, it should be considered as a demonstration SAM reflecting many of the characteristics of a prototype African economy. The SAM is disaggregated in terms of four factors, i.e., unskilled labour, skilled labour, capital and agricultural capital (i.e., land); six categories of households, i.e., rural (landless) workers, rural land owners (small), rural land owners (large), urban low education (and hence relatively low-income), urban high education (high-income)⁵ and capitalists; and enterprises. Six production activities are identified; domestic agriculture, export agriculture, mining, industries, services and public services. Finally, five different commodities are specified: domestic agriculture, export agriculture, mining, industries and services.

⁴Since the expenditure (income) elasticity for household group h and commodity i (ϵy_{hi}) is equal to the ratio of the marginal expenditure propensity (MEP_{hi}) to the average expenditure propensity (AEP_{hi}), it follows that the matrix of marginal expenditure propensities, C_{13} , can be readily obtained once the expenditure elasticities and average expenditure propensities (A_{13}) are known, i.e., since $\epsilon y_{hi} = MEP_{hi}/AEP_{hi}$, $MEP_{hi} = \epsilon y_{hi} AEP_{hi}$.

⁵For example, one could classify "low-education" households as those in which the head of the household had equivalent of a primary education or less; and "high-education" households as those in which the head possessed more than a primary education.

Table 3: Social accounting matrix for archetype African developing country

		Factors				Households						Activities								
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
Factors	Unskilled labour	1											365.5	81.0	38.5	474.2	293.2	267.8		
	Skilled labour	2											4.5	10.0	144.6	107.8	97.7	202.2		
	Capital	3											72.0	30.0	292.4	955.3	567.1	11.9		
	Land	4											361.6	85.0	0.0	0.0	0.0	0.0		
H'holds	Rural workers	5	228.0	0.0	0.0	0.0														
	Rural landowners (small)	6	790.5	0.0	255.9	158.3														
	Rural landowners (large)	7	76.0	141.7	511.8	290.3														
	Urban low education	8	425.7	0.0	85.3	0.0														
	Urban high education	9	0.0	226.7	341.2	0.0														
	Capitalists	10	0.0	198.4	511.8	0.0														
	Enterprise	11			222.7															
Activities	Agriculture	12																	1 038.3	
	Export agriculture	13																	0	
	Mining	14																	0.0	
	Industries	15																	0.0	
	Services	16																	0.0	
	Government	17																	0.0	
Comm.	Agriculture	18				95.0	412.7	271.9	171.6	97.1	32.4		204.8		0.0	323.3	0.0	0.0		
	Exp. Agr.	19				0.0	0.0	0.0	0.0	0.0	0.0			40.0						
	Mining	20				0.0	0.0	0.0	0.0	0.0	0.0		0.0		19.3	43.1	0.0	0.0		
	Industries	21				83.1	402.8	384.2	191.7	242.4	153.1		186.1	30.0	337.5	301.7	143.3	0.0		
	Services	22				57.5	291.0	271.9	141.2	146.1	98.6		0.0		173.6	43.1	247.6	76.5		
	Government	23				12.4	60.1	51.0	26.5	26.4	71.0		25.0	5.0	36.5	81.6	86.1	35.6	85.6	
	Accumulation	24				0.0	36.1	40.8	0.0	53.9	355.1	222.7								
	ROW	25																	759.8	
	Total		1 520.2	566.8	1 928.7	446.6	248.0	1 202.7	1 019.8	531.0	567.9	710.2	222.7	1 219.5	281.0	1 042.4	2 330.1	1 435.0	594.0	1 883.7

Source: Thorbecke and Stifel (1998).

Table 4: Matrix of average expenditure propensities (An) for an archetype African developing economy

		Factors				Households						Enter	Activities						Commodities					Govt	Accum.	ROW
		Unsk L	Skilled L	Capital	Agr Cap	R worker	R own Sm	R own Lg	Urb. Low	Urb High	Capitalist		Agr	Ex Agr.	Mining	Indust.	Service	Pub. Serv.	Agr.	Ex Agr.	Mining	Indust	Service			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
actors	Unskilled labour	1										0.30	0.29	0.04	0.20	0.20	0.45									
	Skilled labour	2										0.00	0.04	0.14	0.05	0.07	0.34									
	Capital	3										0.05	0.11	0.28	0.41	0.40	0.02									
	Agr. Capital	4										0.30	0.30	0.00	0.00	0.00	0.00									
holds	Rural workers	5	0.15	0.00	0.00	0.00																		0.03		
	Rural landowners (small)	6	0.52	0.00	0.13	0.35																		0.00		
	Rural landowners (large)	7	0.05	0.25	0.27	0.65																		0.00		
	Urban low education	8	0.28	0.00	0.04	0.00																		0.03		
	Urban high education	9	0.00	0.40	0.18	0.00																		0.00		
	Capitalists	10	0.00	0.35	0.27	0.00																		0.00		
	Enterprise	11	0.00	0.00	0.12	0.00																				
ivities	Agriculture	12																0.55	0.00	0.00	0.00	0.00				0.16
	Export Agriculture	13																0.00	1.00	0.00	0.00	0.00				0.20
	Mining	14																0.00	0.00	1.00	0.00	0.00				0.45
	Industries	15																0.00	0.00	0.00	0.85	0.00				0.17
	Services	16																0.00	0.00	0.00	0.00	0.65				0.10
	Public services	17																0.00	0.00	0.00	0.00	0.29				0.00
mm.	Agriculture	18					0.38	0.34	0.27	0.32	0.17	0.05		0.17	0.00	0.00	0.14	0.00	0.00					0.00	0.35	
	Exp. Agr.	19					0.00	0.00	0.00	0.00	0.00	0.00		0.00	0.14	0.00	0.00	0.00	0.00					0.00	0.01	
	Mining	20					0.00	0.00	0.00	0.00	0.00	0.00		0.00	0.00	0.02	0.02	0.00	0.00					0.00	0.57	
	Industries	21					0.34	0.33	0.38	0.36	0.43	0.22		0.15	0.11	0.32	0.13	0.10	0.00					0.00	0.06	

Services	22		0.23	0.24	0.27	0.27	0.25	0.14		0.00	0.00	0.17	0.02	0.17	0.13				0.69	0.00		
Government	23		0.05	0.05	0.05	0.05	0.05	0.10		0.02	0.02	0.04	0.04	0.06	0.06	0.05	0.00	0.00	0.03	0.00		
Accumulation	24		0.00	0.03	0.04	0.00	0.09	0.50	1.00											0.25		-0.08
ROW	25															0.40		0.12	0.05			

Table 4, which is derived from Table 3, gives the matrix of average expenditure propensities (A_n) for this archetype African economy. A few examples suffice to show the type of information contained in Table 4. It can be seen that out of total domestic agricultural production unskilled labour receives 30%, capital 6% and agricultural capital 30% (column 12). In turn, total intermediate inputs used in agriculture amount to 32% (column 12). If one were interested in the consumption pattern of rural workers, one could determine from column 5 that 38% of their total income (equal expenditures) was spent on food commodities (agriculture), 34% on manufactured goods and 23% on services. Rural workers' households save nothing and pay only 5% in taxes.

Finally, Table 5 presents the matrix of accounting multipliers for this same archetype economy. Again a few examples can illustrate how this multiplier table can be interpreted. As discussed previously, the endogenous accounts are factors, households, activities and commodities, while the government account, the capital account and the rest of the world are taken as exogenously determined. Thus, if one were interested in the impact of a change in agricultural exports on the whole socioeconomic system, one could read the corresponding multipliers along column 13 of Table 5. In this case x in Equation 3 would reflect a change in agricultural exports and a 100-unit reduction in exports would reduce the incomes of rural workers by 12 units, rural land owners (small) by 68 units, rural land owners (large) by 58 units, urban low-education households by 26 units, urban high-education households by 19 units and capitalists by 24 units (read down column 13 of Table 5). A perusal of Table 5 reveals that changes in different types of exports have very different distributional consequences as the intersection of the activities accounts (columns 12-17) and household income accounts (rows 5-10) shows.

Besides the exogenous shock consisting of a change in exports, one could think of the shock as consisting of subsidies provided directly by the government to certain household groups—say in the form of direct transfers. For example, if the urban low income group were to receive a direct transfer of 100 units, it would result in an increase in domestic agricultural production of 48 units and an even higher increase in the production of industrial goods of 90 units (see the intersection of column 8 and rows 12 and 15, respectively). In this last example, the mechanism leading to the ultimate increase in agricultural production follows the triangular route of Figure 2. If a given household group receives an (exogenous) direct transfer from the government, this will increase their income and allow them to spend additional consumption items. In turn, this increased demand for commodities has to be satisfied through a corresponding increase in production, which leads to a flow of factor earnings (e.g., wages for unskilled and skilled labour) that is next received by the socioeconomic groups possessing those factors of production.

Table 5: Accounting multipliers for the archetype African developing economy

		Factors				Households						Activities							Commodities					
		Unsk. L	Skilled L	Capital	Agr. Cap	R worker	own sm	R own lg	Urb. Low	Urb high	Capist	Enter	Agr.	Ex. Agr.	Mining	Indust.	Services	Ub. Services	Agr.	Ex. Agr.	Mining	Indust.	Services	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	
Factors	Unskilled labour	1	1.5	0.4	0.37	0.5	0.5	0.49	0.5	0.51	0.48	0.23	0.00	0.77	0.81	0.51	0.62	0.65	0.91	0.43	0.81	0.51	0.53	0.69
	Skilled labour	2	0.14	1.12	0.11	0.14	0.14	0.14	0.15	0.15	0.14	0.07	0.00	0.13	0.18	0.29	0.16	0.21	0.48	0.07	0.18	0.29	0.14	0.28
	Capitalists	3	0.57	0.48	1.43	0.57	0.56	0.56	0.58	0.59	0.58	0.29	0.00	0.6	0.69	0.88	0.9	0.92	0.55	0.33	0.69	0.88	0.77	0.77
	Agr. Capital	4	0.14	0.09	0.09	1.13	0.15	0.14	0.13	0.14	0.11	0.05	0.00	0.44	0.48	0.1	0.12	0.1	0.11	0.24	0.48	0.1	0.1	0.1
H'holds	Rural workers	5	0.23	0.06	0.05	0.07	1.08	0.07	0.07	0.08	0.07	0.03	0.00	0.12	0.12	0.08	0.09	0.1	0.14	0.06	0.12	0.08	0.06	0.1
	Rural landowners (small)	6	0.91	0.3	0.41	0.73	0.39	1.38	0.38	0.4	0.36	0.17	0.00	0.64	0.68	0.42	0.49	0.5	0.58	0.35	0.68	0.42	0.41	0.5
	Rural landowners (large)	7	0.35	0.49	0.49	0.95	0.31	0.3	1.3	0.31	0.29	0.14	0.00	0.52	0.58	0.39	0.39	0.39	0.38	0.28	0.58	0.39	0.33	0.37
	Urban low education	8	0.45	0.13	0.17	0.16	0.17	0.16	0.17	1.17	0.16	0.06	0.00	0.24	0.26	0.18	0.21	0.22	0.28	0.13	0.26	0.18	0.18	0.23
	Urban high education	9	0.16	0.53	0.3	0.16	0.15	0.15	0.16	0.16	1.16	0.08	0.00	0.16	0.19	0.27	0.22	0.25	0.29	0.09	0.19	0.27	0.19	0.25
Capitalists	10	0.2	0.52	0.42	0.2	0.2	0.2	0.21	0.21	0.2	1.1	0.00	0.2	0.24	0.33	0.3	0.32	0.31	0.11	0.24	0.33	0.25	0.3	
	Enterprise	11	0.07	0.06	0.17	0.07	0.06	0.06	0.07	0.07	0.07	0.03	1.00	0.07	0.08	0.1	0.1	0.11	0.06	0.04	0.08	0.1	0.09	0.09
Activities	Agriculture	12	0.48	0.31	0.31	0.45	0.51	0.48	0.44	0.48	0.37	0.16	0.00	1.49	0.42	0.32	0.41	0.33	0.37	0.82	0.42	0.32	0.35	0.33
	Export Agriculture	13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.17	0.00	0.00	0.00	0.00	0.00	1.17	0.00	0.00	0.00
	Mining	14	0.02	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.00	0.02	0.02	1.04	0.03	0.01	0.01	0.01	0.02	1.04	0.03	0.01
	Industries	15	0.87	0.75	0.67	0.88	0.86	0.85	0.89	0.9	0.92	0.45	0.00	0.88	0.9	0.95	1.78	0.77	0.75	0.48	0.9	0.95	1.52	0.73
	Services	16	0.44	0.37	0.34	0.45	0.43	0.43	0.46	0.46	0.44	0.22	0.00	0.36	0.4	0.45	0.34	1.47	0.47	0.2	0.4	0.45	0.29	1.1
Public services	17	0.2	0.17	0.15	0.2	0.19	0.19	0.2	0.21	0.2	0.1	0.00	0.16	0.18	0.2	0.15	0.21	1.21	0.09	0.18	0.2	0.13	0.49	
Comm.	Agriculture	18	0.87	0.57	0.58	0.82	0.92	0.87	0.79	0.87	0.68	0.29	0.00	0.89	0.75	0.58	0.74	0.6	0.68	1.49	0.75	0.58	0.63	0.6
	Export agriculture	19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.17	0.00	0.00	0.00	0.00	0.00	1.17	0.00	0.00	0.00
	Mining	20	0.02	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.00	0.02	0.02	0.04	0.03	0.01	0.01	0.01	0.02	1.04	0.03	0.01

Industries	21	1.02	0.88	0.79	1.03	1.01	1.00	1.06	1.05	1.08	0.53	0.00	1.03	1.06	1.12	0.92	0.9	0.88	0.57	1.06	1.12	1.78	0.85
Services	22	0.67	0.58	0.51	0.58	0.65	0.66	0.69	0.7	0.67	0.34	0.00	0.55	0.61	0.68	0.52	0.71	0.72	0.31	0.61	0.68	0.44	1.68

Total factors		2.35	2.09	2.00	2.35	1.35	1.33	1.36	1.39	1.31	0.64	0.00	1.94	2.15	1.78	1.81	1.88	2.05	1.07	2.15	1.78	1.54	1.84
Total labour		1.54	1.52	0.47	0.64	0.64	0.63	0.64	0.66	0.62	0.3	0.00	0.9	0.98	0.8	0.79	0.86	1.39	0.5	0.98	0.8	0.67	0.97
Total institutions		2.35	2.09	2.00	2.35	2.35	2.33	2.36	2.39	2.31	1.64	1.00	1.94	2.15	1.78	1.81	1.88	2.05	1.07	2.15	1.78	1.54	1.84
Total activities		2.00	1.61	1.48	2.00	2.01	1.97	2.01	2.06	1.95	0.94	0.00	2.91	3.08	2.96	2.71	2.79	2.82	1.6	3.08	2.96	2.31	2.66
Total commodities		2.58	2.02	1.87	2.55	2.6	2.54	2.56	2.64	2.44	1.17	0.00	2.48	2.61	2.42	2.21	2.23	2.29	2.37	3.61	3.42	2.88	3.14

A crucial feature of a SAM is that it provides disaggregated information on income distribution across socioeconomic household groups (the row total in Table 3) as well as the factorial sources of income of each household category (i.e., the transaction submatrix T_{21} or coefficient submatrix A_{21} in Table 2). As indicated previously this matrix reflects the resource (factor) endowment of the different household groups. The SAM also reveals the sectoral production origin of factorial income (T_{13} and A_{13} , respectively). This mapping reflects the structure of production and the technology used to produce the different production activities.

Table 6 presents the factorial source of income for each socioeconomic group in the archetype African economy.

Table 6: Factorial source of household income (matrix A_{21} in Table 2)

	Unskilled Labour	Skilled Labour	Capital	Land	Transfers	Total
Rural workers	91.94%				8.06%	100%
Small rural landowners	65.72%		21.28%	13.00%		100%
Large rural landowners	7.45%	13.89%	50.19%	28.47%		100%
Urban low education	80.17%		16.06%		3.77%	100%
Urban high education		39.92%	60.08%			100%
Capitalists		27.94%	72.06%			100%

As we can observe in Table 6, the composition of income of each household group is related to its social classification. The incomes of the rural workers, the small rural landowners and urban low-education group consist mostly of unskilled labour receipts, while large landowners, the urban high education and the capitalist households receive the bulk of their income from capital and land rent.

In Table 7, we present the share of the primary factors in the value added for each branch of production. The agricultural (traditional and export agriculture) and services (service and public service) sectors are mostly intensive in unskilled labour and the industrial (mining and industries) sectors intensive in the capital primary factor. Skilled labour is used more intensively in the public services branch and in the mining branch. As for land, only the agricultural branches share this resource.

It will be seen in the next section, where a CGE is calibrated on the present archetype African SAM and used to simulate among others a trade shock, that the latter affects income distribution through its impact on factor employment. In summary the impact of exogenous shocks is transmitted throughout the channels of the socioeconomic system given by archetype SAM. By studying tables 6 and 7, we can see that a shock affecting the agricultural sectors would have a greater impact on rural households' income than on the capitalists' income.

Table 7: Share of the primary factors in the value added

	Agriculture	Export agriculture	Mining	Industries	Services	Public service
Unskilled Labour	45.48%	39.32%	8.1%	30.85%	30.61%	55.57%
Skilled labour	0.56%	4.85%	30.41%	7.01%	10.2%	41.96%
Capital	8.96%	14.56%	61.49%	62.14%	59.2%	2.47%
Land	45.00%	41.26%				
Total	100%	100%	100%	100%	100%	100%

A final issue that needs to be emphasized is that the SAM by itself, can provide only limited information on poverty. Since the SAM, as such, provides information on the total and average incomes received by the respective household groups, it ignores the intra-group income distribution (or more exactly it assumes implicitly that the intra-group variances are zero). It is only if the intra-group income distribution is known that poverty per se can be determined and analysed. One attempt in that direction is that of Thorbecke and Jung (1996). They developed a multiplier decomposition technique focusing more specifically on the extent to which different production activities affect different household groups' incomes, and ultimately poverty alleviation, and the structural mechanisms and linkages through which an initial rise in a sector's output contributes directly or indirectly to poverty alleviation. The poverty alleviation effects were decomposed into the product of (1) the changes in average incomes received by the various household groups resulting, directly or indirectly, from the growth of a sectors' output; and (2) the poverty-sensitivity effects that, in turn, depend on the respective household groups' poverty elasticities with respect to groups' mean incomes and the intra-group income distributions.

The next sections of this paper proceed to incorporate intra-group income distributions as well as the derivation of the poverty line and the measurement of poverty within a general equilibrium framework.

6. General equilibrium models, income distribution and poverty

Computable general equilibrium (CGE) models have traditionally been used to simulate the impact of exogenous shocks (such as changes in international terms of trade, or a recession in importing countries), as well as changes in policies on the socioeconomic system and, in particular, income distribution. Good examples of such models are those that are built in connection with the OECD research programme to explore the impact of structural adjustment on equity (see, e.g., Thorbecke, 1991, for Indonesia; de Janvry et al., 1991, for Ecuador; Morrisson, 1991, for Morocco). Still an additional model developed in the context of Africa is that of Chia et al. (1994). These models allowed the impact of counterfactual policy scenarios to be simulated on income distribution. Since CGE models are fully calibrated on the basis of an initial year SAM that provides a set of consistent initial

conditions—and the SAM, as such, does not contain information on intra socioeconomic household group income distribution, it follows that conventional CGEs can only simulate the impact of a shock on the representative household in each group. This amounts to the implicit assumption that the variance of income within a group is zero. To the extent that poverty is pervasive and is likely to affect many socioeconomic groups (albeit to different degrees) it appears essential in any analysis of the impact of a shock on poverty to start with information on intra-group income distribution. Increasingly, as more income and expenditure surveys become available, it is possible to generate the within-group income distributions prevailing in the same base year as that of the SAM used to calibrate the general equilibrium model.

There have been some attempts in the literature to postulate given intra-group distributions and assess the impact on poverty through a general equilibrium model. Thus, for example, de Janvry et al. (1991) use both the lognormal and the Pareto distribution functions to depict income distribution of each household group. The authors do not justify why these functional forms are more appropriate than more flexible forms. In Adelman and Robinson (1979), a statistical test is performed on the lognormal, and in some cases the test (skewedness and kurtosis) were not satisfactory. They simply eliminated a socioeconomic group (by aggregation) to circumvent the problem. The income distribution modeling approach and the statistical literature provide evidence that other functional forms might be more appropriate to represent income distribution (see Bordley et al., 1996).

We follow these authors by assessing poverty through a general equilibrium model. However, we differ from their approach in three ways. The first is by proposing a more flexible income distribution function. Second, the intra-group distributions are specified so as to conform to the different socioeconomic characteristics of the groups. Thus, for example, as will be seen subsequently the characteristics displayed by rural landless households contrast markedly with those of large landowner households and yield significantly different distributions. Third, we postulate a poverty line based on a unique and constant basket of basic needs commodities. Since commodity prices are endogenously determined within the model, the monetary value of the poverty line is also endogenously determined. These three innovations help shed more light on the black box pertaining to the behaviour of poverty following a shock.

7. Income distribution, poverty and poverty measurement

In this illustrative case and consistent with the SAM (given in Table 3), we aggregate the households into six groups representative of those living in an archetype African country. The groups are defined as follows: (1) rural households (i.e., the landless), (2) small landowner households, (3) large landowner households, (4) urban low-education households, (5) urban high-education households and (6) capitalist households. To each of these groups we attribute income and demographic characteristics typical of an African economy. These descriptive data are presented in Table 8. As we can observe, the mean income varies from 13.57 for the rural households to 117.72 for the capitalist households. As for the population shares, the small landowner category is the largest group with 36.1% of the total population. In this example the rural household group has the highest headcount ratio with 93.3% of its population below the poverty line, followed by the urban low-education

category with 57.7%. It should be noted that in the great majority of developing countries detailed income and expenditure survey data exist from which the actual intra-group income distributions can be derived.

Table 8: Income and demographic characteristics

	Rural households	Small landowner households	Large landowner households	Urban low-education households	Urban high-education	Capitalist households
Mean income	13.57	27.75	29.91	23.27	41.49	117.72
Maximum income	40.0	50.0	55.0	40.0	60.0	140.0
Minimum income	5.0	10.0	15.0	15.0	20.0	25.0
Population share	0.13	0.31	0.25	0.17	0.10	0.04
% below the poverty line	93.3%	36.1%	19.4%	57.7%	0.5%	0%

In order to analyse and derive poverty by household group, we propose an income distribution formulation corresponding to the characteristics of each household group. This distribution will depend on the minimum and maximum incomes and on the skewedness of the income distribution.⁶ To represent these characteristics into our income distributions, we use the Beta distribution function (Equation 5). Parameters mx and mn are, respectively, the maximum and minimum incomes within a group. As for the parameters p and q , they influence the shape and the skewedness of the distribution.

$$I(y; p, q) = \frac{I(y - mn)^{p-1} (mx - y)^{q-1}}{B(p, q) (mx - mn)^{p+q-1}}$$

$$\text{where, } B(p, q) = \int_{mn}^{mx} \frac{(y - mn)^{p-1} (mx - y)^{q-1}}{(mx - mn)^{p+q-1}} dy \quad (5)$$

⁶A left-skewed distribution can be illustrated as the mode being inferior to the median. A right-skewed distribution is when the mode is superior to the median.

Unlike the lognormal, the Beta function is much more flexible when it comes to the asymmetric forms it can adopt. Contrary to the lognormal, the Beta function can be skewed to the left or to the right and be symmetric. If $p > q$, the distribution is skewed to the left. The mode is situated on the left side of the distribution. As the inequality between p and q increases, the distribution tends to be more leftward skewed. If $q > p$, the distribution becomes skewed to the right. Again, the asymmetry accentuates as the inequality increases. If $q = p$, the function becomes symmetric. These three conditions are true only if the values taken by q and p are larger than unity.⁷

Table 9 presents the parameters assigned to each household category. In each case, we have chosen the parameters so that the income distribution concords with the characteristics of the household groups described in Table 1.⁸

Table 9: Parameters for the Beta Distribution Function

	Rural households	Small landowner households	Large landowner households	Urban low-education households	Urban high-education households	Capitalist households
p	1.3	2.0	3.0	1.8	3.3	6.0
q	4.0	2.5	5.0	3.5	3.0	1.5
mx	40.0	50.0	55.0	40.0	60.0	140.0
mn	5.0	10.0	15.0	15.0	20.0	25.0

These distributions will be used to evaluate the poverty incidence within each group in a general equilibrium framework. Following an external shock on the economy, we assume—albeit arbitrarily—that the intra-group distributions shift proportionally with the change in the mean income (as in previous studies, we consider the variance of each distribution exogenous to the model). Since we are not aware of how the increase (decrease) of income is distributed—i.e., if the increase (decrease) in the mean income is the result of an increase (decrease) in the income of the poorest or an increase (decrease) in the income of the richest—we distribute the gain (loss) to all the households within the group. If the mean income increases by Ψ , the income of each household within a group is raised by Ψ .⁹ The income distribution will thus proportionally shift horizontally following a change in

⁷The Beta can also represent a bimodal distribution. This is the case when p and $q < 1$. For other possibilities of the value taken by p and q , the interested reader is invited to consult Chapter 14 in Johnson and Kotz (1970).

⁸In an applied study based on survey data, parameters p and q should be estimated and mx and mn would represent the maximum and minimum incomes within each household category in the survey.

⁹Note that this implies that within a group the poorer the households the greater their relative income gain in the case of an increase in group average income and the greater their relative income loss in the case of a drop in mean income.

income. Although there are recorded cases of significant changes in intra-group distributions following a shock, as in the case of Indonesia's adjustment process between 1984 and 1987 (see Huppie and Ravallion, 1991), more recent work by Ravallion and Chen (1997) finds that inequality increases as often as it falls during spells of growth in developing countries and that neutrality is actually a defensible first-order approximation. However, it is unlikely that distribution neutrality can be assumed to prevail following shocks leading to negative growth, (such as the Asian financial crisis) and it is unclear even in spells of growth, whether distribution neutrality would be a good first-order approximation in estimating poverty as opposed to inequality. As stressed by Dervis et al. (1982), the complete endogenization of intra-group income distributions following shocks still remains the biggest challenge in studying income distribution in a general equilibrium context.

The procedure described above allows us to compare the poverty levels obtaining in the post-simulation case with those prevailing in the pre-simulation case using Foster, Greer and Thorbecke's (FGT) P_α measures (Foster et al., 1984). The FGT P_α class of additively decomposable poverty measures allows us to measure the proportion of poor in the population (the headcount ratio), and also the depth and severity of poverty. The P_α measure expressed in terms of the Beta distribution given in Equation (5) becomes:

$$P_\alpha = \int_{mn}^z \left(\frac{z-y}{z} \right)^\alpha I(y, p, q) dy \quad (6)$$

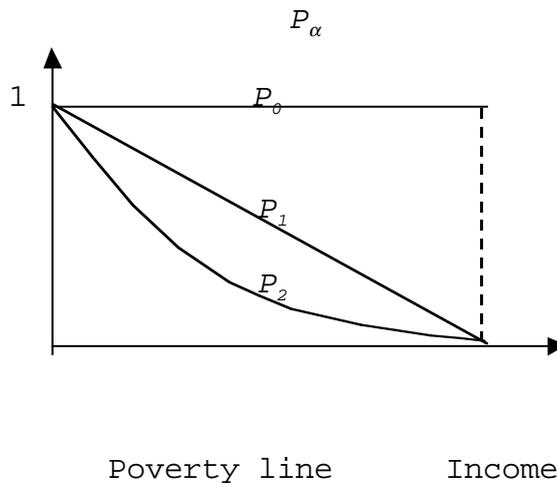
where α is a poverty-aversion parameter, z is the poverty line, mn is the minimum (intra-group) income, and p and q are parameters of the Beta function as defined earlier.

When $\alpha = 0$, the headcount ratio is derived from Equation 6. In this case, the P_α yields the proportion of the population within a group below the poverty line. With $\alpha = 1$, the relative importance accorded to all individuals below the poverty line is proportional to their incomes and we have the income-poverty gap. As α increases, more importance is given to the shortfalls of the poorest households and the measure becomes more distributionally sensitive; society becomes more averse to poverty. In this illustrative study, we set $\alpha = 2$ to interpret this last case, which assumes that each poor household is assigned a weight equal to its shortfall from the poverty line.

We can summarize the three types of measures derived from the P_α class with the help of Figure 4, which shows how the relationship between individual poverty and the standard of living varies across the different values of α (Ravallion, 1994). Figure 4 depicts the P_α measure in relation to income for one individual. For P_0 , the relation with income is constant. The measure accords the same weight to the richest of the poor as it does to the poorest of the poor. Thus, the sum of each individual's P_0 is simply the headcount ratio. The second measure, P_1 , has a linear and decreasing

relation with income. Since the income gap (income - poverty line) grows larger, more importance is given to the poorest and less to the richest in the poverty measure. The last measure quantifies the aversion of the society towards poverty: P_2 is strictly convex in income. This implies that the increase in "our measured poverty due to a fall in the standard of living will be greater the poorer you are" (Ravallion, 1994: 48).

Figure 4: Individual poverty measures



Source: Ravallion (1994).

The poverty line itself (z in Equation 6) is determined endogenously within the CGE model. We postulate that the poverty line is determined by a basket of quantities of commodities reflecting basic needs (BN) consistent with Ravallion's (1994) approach to estimating absolute poverty. We denote this basket as $\bar{\omega}^p_{com}$. This basket remains invariant from one simulation to another and applies to all households regardless of group membership. In turn, the monetary poverty line is obtained by multiplying the BN commodity basket by the respective prices (Pq_{com}) and aggregating across commodities:

Monetary Poverty Line:

$$\sum_{com} \bar{\omega}_{com}^p Pq_{com}$$

Since commodity prices are endogenously determined within the model, so is the nominal value of this basket, i.e., the poverty line. If commodity prices rise following an external shock, the poverty line will increase (shift to the right) and poverty will rise, all things being equal.

The demand system specified in our model is based on the linear expenditure system (LES):

$$C_{hcom} = \frac{Pq_{com}\bar{\omega}_{h,com} + \beta_{h,com}^c \left(CH_h - \sum_{com} \bar{\omega}_{com} Pq_{com} \right)}{Pq_{com}} \quad (7 = 28 \text{ in App. A})$$

where $C_{h,com}$ = the demand for commodity com by household group h
 $\bar{\omega}_{h,com}$ = the basket of committed (minimal) consumption in volume terms for the commodities specific to household group h
 C_h = disposable income of household group h
 Pq_{com} = the price of com
 $\sum_{com} \bar{\omega}_{h,com} Pq_{com}$ = the monetary value of the committed (minimum) consumption specific to household group h.

This demand system implies that each socioeconomic group has its own perception of the minimal commodity basket that it needs to satisfy, consistent with the socioeconomic characteristics and the overall standard of living of the group. Clearly, this minimum basket is bound to be different for the high-income capitalist group than the low-income rural households. Hence the first term on the right-hand side in the numerator of Equation 7 represents the amount needed to satisfy this household-specific minimum consumption requirement of commodity com. In turn, the second term in the numerator represents the proportions or marginal expenditure propensities ($\beta_{h,com}^c$) of discretionary income ($CH_h - \sum_{com} \bar{\omega}_{h,com} Pq_{com}$) to be spent on each respective commodity. It can be seen that if this

last term is zero (i.e., there is no discretionary income) each household group consumes a quantity of each commodity corresponding exactly to its household-specific postulated minimum.

It is essential to grasp clearly the distinction between the poverty BN basket that applies to all households—regardless of group membership—and is defined at the level of the society, and the LES demand system that specifies a group-specific consumption level for each commodity that is intractable downward. Each group is assumed to behave lexicographically in such a way that it first satisfies its minimum consumption of the respective commodities.

8. Model description

The general equilibrium model presented here is inspired by the framework of the Decaluwé et al. (1995) models, which in turn follow the guidelines put forward by the Shoven and Whalley models (1972, 1984). This model represents a small open economy characterizing a developing country that has no influence on the international markets.

The model is described as a six-sector model (traditional agriculture, export crop, mining, industry, service and administrative service). The import competing sectors are industry and traditional agriculture. The export sectors are represented by mining and export crops. Land, agricultural capital, capital, unskilled labour and skilled labour are the five primary factors of production used by the

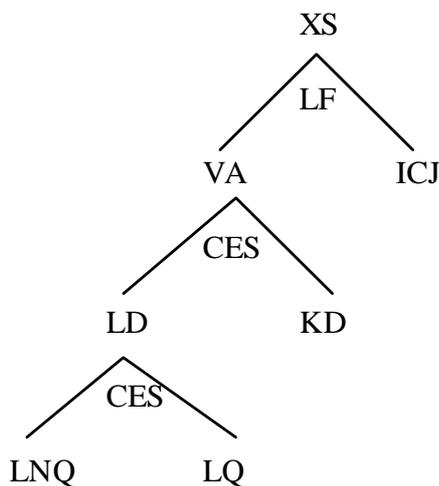
activities. As mentioned in the first part of the paper, households are aggregated into six groups (rural households, small landowner households, large landowner households, urban low-education households, urban high-education households and capitalist households). The geographical location of a household and the origin of their income or occupation and other socioeconomic characteristics define the groups. For example, a rural household has the characteristics of living in rural areas and being endowed exclusively with unskilled labour (and thus being landless). The full model is given in Appendix A, and described below.

Production and employment

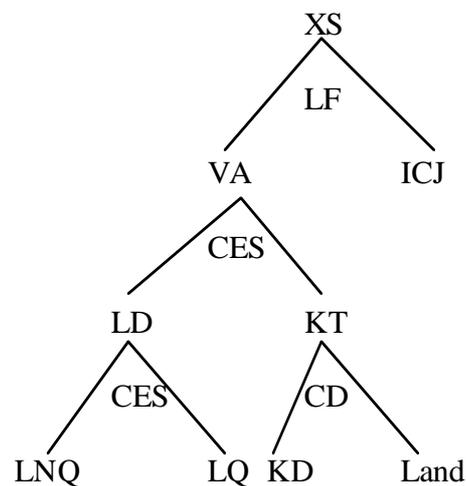
The multi-level cascading specification of the production process is shown graphically in Figure 5. Production activities are broken down into two agricultural activities and four non-agricultural activities. At the highest level of aggregation there are two aggregate inputs, i.e., value added (VA) and intermediate inputs (ICJ), which combine in fixed proportions (Leontief function) to produce sectoral output (XS). At the next level of aggregation value added (VA) is a CES function of composite labour (LD) and composite capital (KD) for the non-agricultural activities and composite labour (LD) and composite agricultural capital (KT) for the two agricultural sectors. Finally, at the lowest level of aggregation composite labour is a CES function of skilled labour (LQ) and unskilled labour (LNQ) and composite agricultural capital is derived from a Cobb-Douglas function combining agricultural capital (KD) and land. Thus, the industrial activities (industry and mining) and service activities (services and administrative services) have a slightly different value-added structure than that characterizing agriculture.

Figure 5: Multi-level production system

Nonagricultural activities



Agricultural activities



This hierarchical multi-level and nested specification allows substitution among primary factors (different labour skill categories and different types of capital in the case of agricultural activities) in the production of the respective activities in response to changes in the relative prices of the factors. The extent of substitutability depends on the magnitude of the respective elasticities of substitution. The production block of the model is given in equations 1-17 in Appendix A.

Labour market

There are two labour markets, for skilled and unskilled workers, respectively. Total supply of these two skill groups is given exogenously and full employment is assumed to prevail. The wage rates for the two labour skills are endogenously determined so as to equate labour supply to labour demand. Labour is assumed to be mobile among the different production activities.¹⁰

Incomes and savings

The households receive their income from primary factor payments and transfers from the government. From this income, we can derive the disposable income by subtracting the direct taxes collected by the government. Savings and total consumption are then specified as fixed proportions of disposable income. A fixed share of capital and land remuneration determines the firms' income. In turn, firms' savings are a fixed share of their income. Government revenue is generated from direct taxes collected on household income, indirect taxes on domestic goods and taxes levied on imports. Government savings are obtained from the difference between government income and expenditures. In turn, government expenditures consist of transfers made to other agents and public consumption. The income and savings block is given in equations 18-26 of Appendix A.

Demand

The demand system—as described previously—that is adopted here is the Stone-Geary linear expenditure system (LES), which is supposed to reflect the household utility functions. As Equation 28 in Appendix A (and Equation 7 in text) shows, a household-specific minimum consumption bundle is postulated where $\bar{\omega}_{h,com}$ represents the minimum quantity of each of the five commodities.¹¹ Multiplying the basket of commodities by respective prices yields the committed (i.e., non-discretionary) income for each group.

¹⁰In further research, the specification of the labour market functioning could allow for sectoral segmentation (e.g., the impact of unions in some sectors) and the possibility of unemployment. Since informal activities are important, these activities, in both the urban and rural areas, can be captured in the base year SAM and the movement of workers between formal and informal activities traced in the model.

¹¹The minimum consumption of a good by a given household group has been derived using the Frisch parameter and the income elasticity. For a detailed presentation, the interested reader is invited to consult Dervis et al. (1982).

Total intermediate demand for a given commodity aggregates the input requirements for that commodity by the various production activities (Appendix equation 31). The investment demand for a good is presented as a fixed proportion of total investment (Appendix equation 32).

Foreign trade

In this model, we follow the Armington (1969) approach by supposing an imperfect substitutability between domestic and imported goods (Appendix equation 45). As for exports, a constant elasticity of transformation characterizes the relative facility of a producer to switch between markets. Following a change in the relative price of domestic and export goods, the producer is able to switch between the domestic and export markets to a degree expressed by the elasticity of transformation (Appendix equation 44). The exogenous current account balance representing the flow of foreign savings (Appendix equation 47) is presented as the difference between the import value and export value.¹²

Equilibrium conditions

Three equilibrium conditions are respected in the model. The first condition implies the equilibrium between the demand for primary factors and its supply, namely one market for skilled labour, one for unskilled labour and one for agricultural capital. There is no market clearing condition for non-agricultural capital and land since they are immobile. The second condition dictates the equilibrium between total investment and total savings. The third equilibrium condition respects the Walrasian framework. The domestic demand for each good is equal to its corresponding supply.

Closure

Since the economy has no impact on international markets, the world prices of import and export are exogenous to the model. The current account balance and the nominal exchange rate are also exogenous to the model. The predetermined current account balance (i.e., foreign savings) has to equal the import surplus (Appendix equation 47). Furthermore, government consumption and its transfers to households are exogenous. As a last closure condition, the primary factor supplies are all exogenous to the model. In this model, both types of labour and agricultural capital are mobile among sectors. Land and capital are specific to each production activity. From Appendix A it can be seen that the model is just identified containing as many endogenous variables as (independent) equations (i.e., 223).

¹²In this archetype economy, there are no transfers coming from or going to the rest of the world.

9. Simulation results

Two simulations are performed on the model's base year equilibrium. The first is a reduction in the world price of the agricultural export crop on the international market and the second is an import tariff reform. The results of these two simulations are summarized in tables 10 and 11. In what follows, we discuss the effects of these simulations on the whole socioeconomic system and how they ultimately affect the household income distribution and poverty based on the P_{-} measures.

Simulation 1: Fall in world price of export crop

The first simulation consists of implementing a 30% fall in the world price of the agricultural export crop. This reduction has a direct repercussion on the exports of agricultural sector. Real output of this sector declines by 35.75%. The primary factors (i.e., skilled and unskilled labour) employed by this sector during the base run are now reallocated to other activities. Consequently, this reallocation increases the production of the other sectors. However, the nominal GDP at factor price falls by 5.88% and the real GDP decreases only marginally by 0.23%. This fall is linked to the decrease in the primary factor prices. In the case of agricultural land, the factor price drops by 55.80%. Land being the primary factor used intensively in agricultural production, a shock on this sector depresses the rate of return on land. This effect is amplified by the constraint that land is specific to each agricultural activity. No reallocation is allowed for this factor, only the rate of return adjusts to the shock. The return on agricultural capital also falls considerably following a decrease in the world price of the export crop (-18.62%). With regard to the other two mobile primary factors, the wages of the unskilled and skilled labour also fall by 6.56% and 3.83%, respectively. Since skilled labour represents 39% of the value added in export agriculture versus 5% for skilled labour, the variation in the first matters most. However, the effects on wages are attenuated by the mobility of the two types of labour among sectors. Because of the assumption of full employment in the two labour markets the workers released from export agriculture are absorbed in other activities.

The drop in factor prices translates directly into a fall in household nominal income. As we can observe in Table 11, the mean nominal income of each household group decreases. The two household categories affected the most by the shock are the small and large landowner households with a fall in their incomes of 6.93% and 6.91%, respectively. This is a consequence of the drastic drop in the rate of return on land and agricultural capital; endowment of these two factors represents 34.3% of the small land owners' household income and 78.7% of the large landowners' household income.

As for the rural household and the urban low-education households, the decline in their nominal income is mainly a consequence of the reduction in the unskilled wage rate since the share of their total income originating from unskilled labour salary represents 91.9% and 80.2% of total income, respectively. The earnings of the urban high-education households and the capitalist households also fall by 4.09% and 4.14%, respectively.

The decline in each household category's nominal mean income is presented by a horizontal shift to the left of the income distribution, as shown in figures 6a through 6f.

Table 10: Simulation results

Variables	Base Level	Simulation 1: 30% decrease in the world price of export of agricultural crop (% change from base)	Simulation 2: 50% decrease in import tariff (% change from base)
GDP nominal	4462.30	-5.88	-1.56
GDP real	4462.30	-0.230	0.551
X _s _{agriculture}	1219.50	3.289	-1.034
X _s _{export agriculture}	281.00	-35.748	2.70
X _s _{mining}	1042.40	2.243	2.70
X _s _{industry}	2330.10	0.567	-1.37
X _s _{service}	1435.90	1.189	-0.18
X _s _{administrative service}	594.00	1.170	0.72
Y _g	679.00	-5.10	-12.54
Y _{dh} _{rural (nominal)}	235.60	-6.03	-1.21
Y _{dh} _{small landowner (nominal)}	1142.60	-6.93	-1.46
Y _{dh} _{large landowner (nominal)}	968.80	-6.91	-1.69
Y _{dh} _{urban low income (nominal)}	504.50	-5.94	-1.31
Y _{dh} _{urban high income (nominal)}	539.50	-4.09	-1.67
Y _{dh} _{capitalist (nominal)}	639.20	-4.14	-1.65
Ex _{agriculture}	181.20	9.94	1.96
Ex _{export agriculture}	231.00	-37.97	3.24
Ex _{mining}	535.00	5.87	2.27
Ex _{industry}	195.00	6.36	2.47
Ex _{service}	110.00	6.45	1.75
M _{agriculture}	759.80	-5.20	0.62
M _{industry}	296.60	-5.94	8.66
M _{service}	100.00	-4.67	-0.49
L _q _{agriculture}	2.250	0.513	-1.481
L _q _{export agriculture}	5.000	56.789	5.128
L _q _{mining}	72.300	5.217	-3.435
L _q _{industry}	53.900	-1.080	-0.141
L _q _{service}	48.850	0.521	2.088
L _q _{administrative service}	101.100	-0.610	1.302
LNQ _{agriculture}	487.333	3.744	-1.892
LNQ _{export agriculture}	108.000	-55.400	4.690
LNQ _{industry}	51.333	8.600	-3.837
LNQ _{mining}	632.267	2.100	-0.557
LNQ _{service}	390.933	3.752	1.663
L _q _{administrative service}	357.067	2.585	0.880
P _m _{service}	1.00	-	-
P _q _{agriculture}	1.06	-3.65	-4.01
P _q _{export agriculture}	1.11	-18.08	-2.71
P _q _{mining}	1.08	-6.54	-6.80

Pq_{industry}	1.07	-4.65	-3.71
Pq_{service}	1.06	-4.77	-1.07
wq	2.00	-3.83	-1.69
wnq	0.75	-6.56	-1.31
Ra	1.00	-18.62	-1.26
$r_{\text{agriculture}}$	1.00	-3.13	-3.04
$r_{\text{exportagriculture}}$	1.00	-55.80	2.99
r_{mining}	1.00	0.72	-4.76
r_{industry}	1.00	-4.78	-1.81
r_{service}	1.00	-3.38	0.18
$r_{\text{administrative service}}$	1.00	-4.37	-0.52

Figure 6: Effect of a 30% reduction in the world export price of the export agriculture crop on income distribution

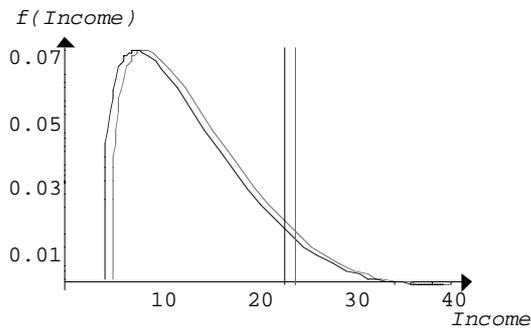


Figure 6a : Income distribution rural households

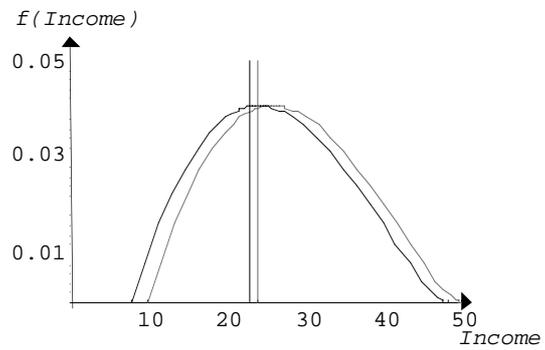


Figure 6b : Income distribution small landowner households

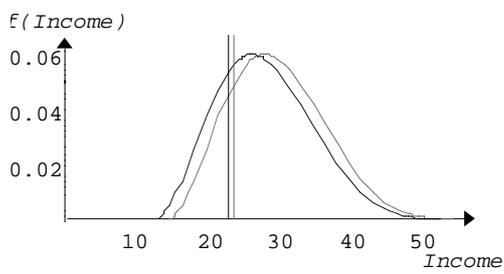


Figure 6c : Income distribution large landowner households

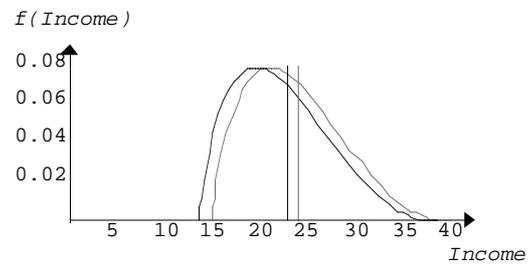


Figure 6d : Income distribution urban low education households

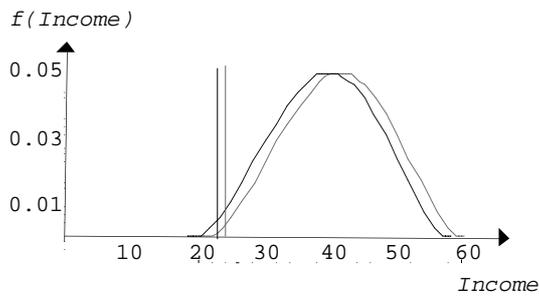


Figure 6e : Income distribution urban high education households

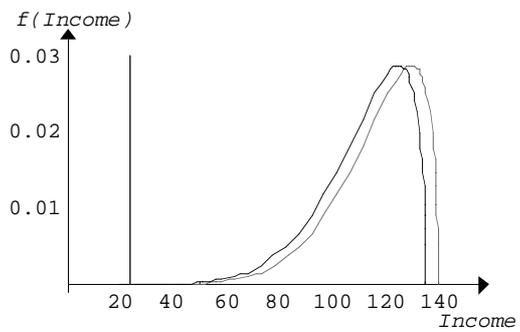


figure 6f : Income distribution capitalist households

Since the prices of the commodities are endogenously determined so is the new monetary poverty line

$$i.e. \sum_{com} \bar{\omega}_{com}^P P q_{com}$$

Hence in figures 6a-6f the new post-simulation poverty line is drawn next to the pre-simulation line. The changes in the poverty line are presented in Table 11 (see bottom panel). For the first simulation the poverty line decreases by 4.4%. This reduction is the consequence of a fall in the consumption prices of the basket of basic needs, which determines the poverty line. With the post-simulation distribution and a new poverty line, we can use the P_{α} class to estimate the effects on poverty.

The headcount ratio (P_0) increases for all household groups except the rural households. Rural households display, by far, the highest headcount ratio, with 92.9% of the population below the poverty line. Compared with the base year, this represents a 0.4% improvement in the headcount ratio. Rural households constitute the only group enjoying a reduction, in poverty. This is explained by the poverty line reduction which dominates the reduction in nominal income of this specific household group. We find the highest relative increase in P_0 among the urban high-education households; the ratio increases from 0.5% to 0.8% following the fall in the price of the export crop.

Table 11: Poverty measures for the base year and simulations

		Rural house- holds	Small landowner household	Large landowner low- education	Urban high- household	Urban households	Capitalist	P_{α}^{social}
alpha=0	base	0,933	0,361	0,194	0,577	0,005	0	0,380
	Simulation 1	0,929 (-0,4%)	0,396 (9,7%)	0,245 (26,4%)	0,600 (4,0%)	0,008 (60,0%)	0	0,407 (7,2%)
	Simulation 2	0,923 -1,0%	0,348 -3,7%	0,183 -5,4%	0,546 -5,3%	0,005 -3,1%	0	0,367 -3,4%
alpha=1	base	0,443	0,078	0,021	0,093	0,00019	0	0,104
	Simulation 1	0,454 2,5%	0,096 22,9%	0,032 50,3%	0,106 13,7%	0,00038 138,6%	0	0,116 11,4%
	Simulation 2	0,434 -2,0%	0,076 -3,3%	0,020 -5,2%	0,086 -7,8%	0,00001* 1,0%	0	0,100 -13,2%
alpha=2	base	0,249	0,024	0,004	0,020	41,5	0	0,045
	Simulation 1	0,263 5,7%	0,033 37,1%	0,006 77,2%	0,025 23,7%	39,9 -4,1%	0	0,051 13,8%
	Simulation 2	0,243 -2,4%	0,024 -2,8%	0,003 -4,9%	0,018 -9,8%	40,8% -1,7%	0	0,043 -3,1%
Mean Income	base	13,6	2,8	29,9	23,	24,0	117,7	
	Simulation 1	12,7 -6,0%	25,8 -6,9%	27,8 -6,9%	21,9 -5,9%	23,0 -4,4%	112,9 -4,1%	
	Simulation 2	13,4 -1,2%	27,3 -1,5%	29,4 -1,7%	23,0 -1,3%	23,3 -3,0%	115,8 -1,7%	
Poverty Line	base	24,0	24,0	24,0	24,0	24,0	24,0	
	Simulation 1	23,0 -4,4%	23,0 -4,4%	23,0 -4,4%	23,0 -4,4%	23,0 -4,4%	23,0 -4,4%	
	Simulation 2	23,3 -3,0%	23,3 -3,0%	23,3 -3,0%	23,3 -3,0%	23,3 -3,0%	23,3 -3,0%	

*Results are identical to the base year data due to rounding of the figures

As opposed to the headcount measure, the poverty gap increases for all households. The rural households display the smallest relative increase in the income gap measure (2.5%) of all the household groups. The same holds true for P_2 ; the rural households reveal the smallest relative increase and the urban high-income households the highest.

In Table 11, an aggregate societal poverty corresponding to each of the three classes of P_α 's, i.e., where $P_\alpha^{social} = \sum_h n_h P_\alpha^{household}$, where n_h is the household population share in the total population.

As we can clearly observe the societal poverty measures of the three classes rise with a decline in the world price of the country's export crop.

In most cases above the effects pertaining to the downward shift in the poverty line do not counter balance (compensate for) the negative nominal mean income effect. The only case for which the downward shift in the poverty line is proportionally greater than the income effect is the P_0 of the rural households. Only in this case does poverty fall following the negative trade shock.

Simulation 2: Import tariff reform

The second simulation consists of implementing a 50% reduction in import tariffs on all imports. This policy reduces the domestic price of imports competing with the traditional agriculture (-5.06%) and the industrial (-10.01%) sectors.¹³ With a fall in the import price, the agents substitute domestic products for imported goods. This in turn leads to a decline in the consumption price of the domestically produced goods originating in the protected traditional agriculture and industrial sectors. The fall in the demand for these two domestically produced goods and the increased competition redirects some of the output towards the export market. Exports of traditional agriculture increase by 1.96% and the exports of the industrial branch by 2.47%. Since increased exports did not completely compensate for the fall in domestic demand, output of these activities falls by 1.03% and 1.37%, respectively. The resources released by this fall in total output in traditional agriculture and industry move to other more efficient activities except for the mining sector. The latter undergoes a slight reduction in output (1.37%) as a consequence of the combined fall of the intermediate demand for the mining product coming from the industrial production and the investment demand for the mining product.

With the fall in almost all of the primary factor prices, the nominal GDP at factor cost decreases by 1.56% but real GDP increases marginally by 0.55%. The fall in the factor prices also has a negative effect on the households' nominal incomes. The large landowners' households suffer the largest decrease in mean income (1.69%) and the rural households suffer the least decrease (1.21%).

¹³One could imagine that the government was protecting a key domestic food crop (e.g., rice) and following a strategy of import substitution industrialization.

As in the first simulation, the post-simulation income distributions are plotted and compared to the base distributions in Figure 7. Again, the fall in the consumption price of all the goods reduces the poverty line (which shifts to the left). The corresponding new poverty levels are presented in Table 11.

In this simulation all headcount ratios improve following a fall in the import tariff (capitalist households display no poverty under either regime). Thus, the shift in the monetary poverty line more than compensates for the fall in mean incomes. With a P_0 of 0.0183, the large landowner households undergo the highest relative decrease in poverty with a fall of 5.4% (of course, the absolute change is small). The smallest relative fall of the P_0 measure is ascribed to the rural households (1.0%).

As with the headcount ratio, the poverty gap measure falls for all households. However, these improvements are relatively less important for the rural households and the urban high-income households (-1.1%) than for the urban low-education households (-7.8%).

All household categories except the urban high-income group benefit from an alleviation of poverty when measured with P_2 . The poverty level for the urban high-education group rises by 1.0%, while the urban low-education group has the largest relative all in poverty (-9.8%).

As for the social poverty levels presented in Table 6, the headcount ratio and the poverty gap decline by 3.4% and 13.2%, respectively. Here, the reductions in import tariffs are beneficial to the alleviation of social poverty. The same is true when society has a greater aversion towards poverty. In this case, the social poverty level declines slightly by -3.1%.

Figure 7: Effect of a 50% reduction in import tariffs on all imports

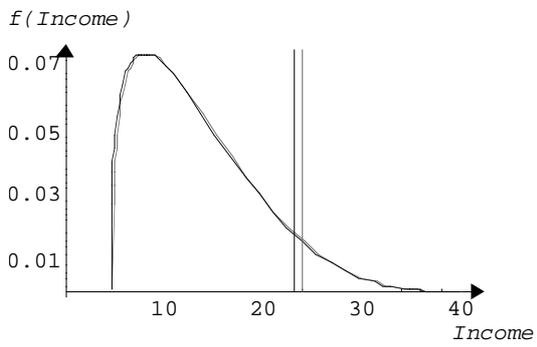


Figure 7a : Income distribution rural households

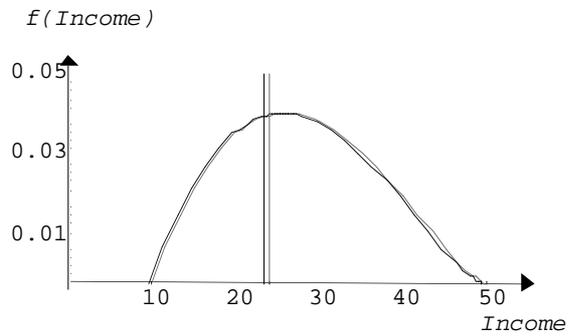


Figure 7b : Income distribution small landowner households

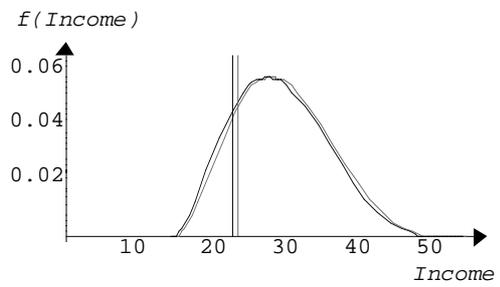


Figure 7c : Income distribution large landowner households

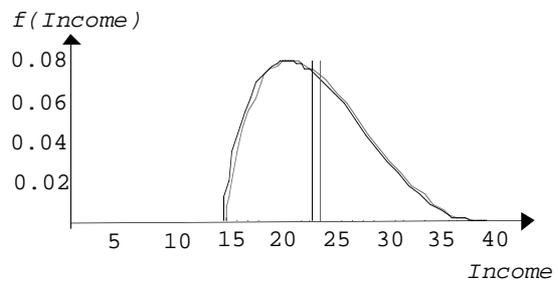


Figure 7d : Income distribution urban low education households

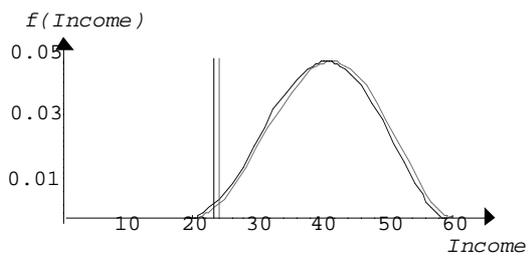


Figure 7e : Income distribution urban high education household

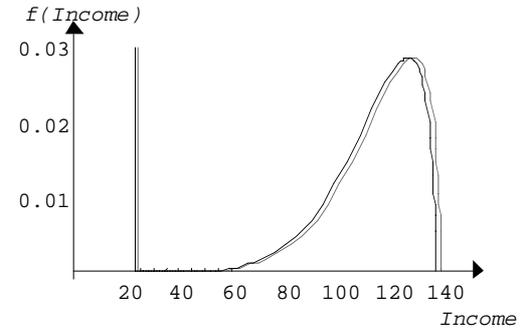


Figure 7f : Income distribution capitalist households

10. Conclusions

This paper showed how the analysis of income distribution and poverty can be incorporated into the SAM and CGE methodology. On the basis of a socioeconomic structure reflecting an archetype African economy a corresponding SAM and CGE were built.

The major contribution of this paper is that the model is specified in such a way that the impact of an exogenous shock on poverty can be simulated. The CGE model takes as its point of departure the initial intra-group income distributions for the six different household categories. By postulating a relatively flexible Beta distribution the parameters of that distribution are chosen so as to conform to and reflect the socioeconomic characteristics of each household category in the model. The poverty line is defined as the cost of a basket of basic needs commodities. Since the basket itself in quantitative terms remains invariant (consistent with the notion of absolute poverty) and prices are endogenously determined within the model, so is the monetary poverty line. The demand system adopted in the CGE model is a variant of the linear expenditure system. Demand functions are specified for each socioeconomic household group and for each commodity. The form of these functions is that they contain a subjectively derived minimum commodity basket specific to each household group and reflecting the socioeconomic characteristics of each group and its standard of living and preferences.

Starting with the initial intra-group income distributions the model simulates the effects of two different shocks (a fall in the price of exports and an import tariff reform) on the average income levels of the household group and assumes that the initial distributions shift horizontally (either to the left or to the right) as mean incomes fall or rise, respectively.

This procedure yields the post-simulation within-group income distributions, which can then be confronted with the new endogenously derived poverty line to measure the resulting poverty. Using the FGT additively decomposable class of poverty measures. In this way a comparison can be made of the incidence of poverty in the pre- and post- simulation situations.

In conclusion, the approach followed in this paper has gone part of the way in endogenizing the effects of exogenous shocks on poverty within a general equilibrium framework. It is hoped that this paper will encourage researchers to analyse and explain more deeply the mechanisms affecting the shape of intra-group income distributions following a shock. A better understanding of those mechanisms would reduce the arbitrariness of assuming—as we do here—that those distributions shift horizontally so that every individual within a household group receives an addition (or, alternatively, a reduction) in income equal to the difference between the post- and pre-simulation average income of that group.

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Appendix A: General equilibrium model of archetype African economy

1. List of symbols

Indexes

Activities:	agr	Agriculture
	agex	Rent agriculture
	min	Mining
	ind	Industry
	ser	Services
	as	Administrative services

Commodities:	agr	Agriculture
	agex	Rent agriculture
	min	Mining
	ind	Industry
	serv	Compositeservices (as, ser)

Sets:	I	{agr, agex, min, ind, ser, as}	Production activities
	AG	{agr, agex }	Agricultural activities
	IN	{min, ind, ser, as }	Non-agricultural activities
	MER	{agr, agex, min, ind, ser, }	Exporting activities
	COM	{agr, agex, min, ind, serv }	Commodities

Households:	mrur	Rural households (landless)
	mls	Small agricultural land owners households
	mll	Large agricultural land owners households
	muli	Low-income urban households
	muhi	High-income urban households
	cap	Capitalist households

H {mrur, mls, mll, muli, muhi, cap }

Parameters

CES Elasticity of substitution of VA	σ
CES Substitution parameter of VA	ρ^i
CET Distributive share of VA	δ^i
CES Scale parameter of VA	B_i
CES Elasticity of substitution of LD	σ_l^i
CES Substitution parameter of LD	ρ_l^i
CET Distributive share of LD	δ_l^i
CES Scale parameter of LD	B_l^i
Cobb-Douglas elasticity for KT	α^{ag}
Cobb-Douglas scale parameter for KT	A_{ag}^d
Cobb-Douglas elasticity for Dserv	α^d
Cobb-Douglas scale parameter for Dserv	A^d
Share of commodity I in householdh consumption	$\beta_{h,com}^c$
Basic poverty needs basket in quantitative terms	ω_{com}^p
Minimal consumption (in quantity) of commodity	

by householdgroup h	h, com
Share of commodity in public consumption	β_{com}^g
Householdmarginal propensity to save	mps_h
Share of commodity in total investment	β_{com}^x
Activity i's share in total production	β_k^i
Householdshare of capital income	λ_t^h
Householdshare of land income	λ_h^h
Householdshare of skilled labour income	λ_h^q
Householdshare of unskilledlabour income	λ_h^{nq}
Input-output coefficients	$aij_{com,j}$
Share of value added in total output	v_i
Householdincome tax rate	tyh_h
Indirect tax rate	tx_i
Import duty rate	tm_{com}
CET scale parameter	B_{mer}^T
CET transformation parameter	ρ_{mer}^T
CET distributive share	δ_{mer}^T
CET elasticity of transformation	σ_{mer}^T
CES scale parameter	B_{com}^S
CES substitution parameter	ρ_S^{com}
CES distributive share	δ_S^{com}
CES elasticity of substitution	σ_{com}^S

Endogenous variables

Price

Wage rate	w_i
Skilled wage rate	wq
Unskilledwage rate	wnq
Value added price	Pva_i
Producer price	P_i
Rate of return on capital	r_{in}
Rate of return on agricultural capital	ra
Rate of return on land	rt_{ag}
Price of compositecapital	PK_{ag}
Price of commodities and services	$Pdi+serv$
Price of composite commodities	Pq_{com}
Domestic price of imports	Pm_{com}
Domestic price of exports	Pe_{mer}
Producer price index	$Pindex$

Production

Value added	VA_i
total production by activity	XS_i

Factors

Agricultural compositecapital	$KTag$
Agricultural capital demand	KD_{ag}
Skilled labour demand	LQ_i
Unskilledlabour demand	LNQ_i
Composite labour demand	LD_i

Demand

Total household consumption	CH_h
Household consumption of commodity com	$C_{h,com}$
Total consumption of commodity i	CT_{com}
Total investment	IT
Consumption of commodity i for investment uses	INV_{com}
Intermediate demand of commodity i	$INTD_{com}$
Intermediate consumption of commodity com by activity j	$ICJ_{com,j}$
Imports (cif vol)	M_{com}
Exports (fob vol)	$EXmer$
Domestic demand for domestically produced commodities	$Di-serv$
Domestic demand for composite commodities	Q_{com}

Income and saving

Total household income	YH_h
Household disposable income	YDH_h
Firm income	YF
Government revenue	YG
Household savings	SH_h
Firm savings	SF
Government savings	SG
Indirect taxes	TXS_i
Revenue from import duties	TXM_{com}

Number of endogenous variables

223

Exogenous variables

Unskilled labour supply	$LSNQ$
Skilled labour supply	LSQ
Non agricultural capital by activity	$KDin$
Agricultural capital stock	KA
Land	$LAND_{ag}$
Nominal exchange rate	e
World price of exports (in foreign currency)	Pwe_{com}
World price of imports (In foreign currency)	Pwm_{com}
Current account balance	CAB
Government transfer payments to household	TGH_h
Public consumption	CG

2. Equations

Production

$$XS_i = \frac{VA_i}{v_i} \quad (1)$$

$$ICJ_{com, j} = a_{ijcom, j} XS_j \quad (2)$$

Non-agricultural activities

CES between composite labour and capital

$$VA_{in} = B_{in} \left[\delta_{in} K_{in}^{-\frac{1}{\rho_{in}}} + (1 - \delta_{in}) LD_{in}^{-\frac{1}{\rho_{in}}} \right]^{\rho_{in}} \quad (3)$$

Labor demand derived from 3.

$$LD_{in} = \left(\frac{VA_{in}}{B_{in}} \right)^{\rho_{in}} \left[(1 - \delta_{in}) + \delta_{in} \left[\frac{\delta_{in} W_{in}}{(1 - \delta_{in}) r_{in}} \right]^{\sigma_{in} - 1} \right]^{\frac{1}{\rho_{in}}} \quad (4)$$

CES between skilled and unskilled labour

$$LD_{in} = B_{in}^i \left[\delta_{in}^i LNQ_{in}^i + \left(1 - \delta_{in}^i \right) LQ_{in}^i \right]^{\frac{1}{\rho_{in}^i}} \quad (5)$$

Labour demand derived from 5.

$$LQ_{in} = \left(\frac{LD_{in}}{B_{in}^l} \right) \left[\left(1 - \delta_{in}^l \right) + \delta_{in}^l \left[\frac{\delta_{in}^l wq}{(1 - \delta_{in}^l)wnq} \right]^{\sigma_{in}^l - 1} \right] \left(\frac{1}{p_{in}^l} \right) \quad (6)$$

Factor prices

$$nn = \frac{Pv_{in}VA_{in} - winLD_{in}}{KD_{in}} \quad (7)$$

$$win = \frac{wqLQ_{in} + wnqLNQ_{in}}{LD_{in}} \quad (8)$$

Agricultural activities

CES between composite labour and capital

$$VA_{ag} = B_{ag} \left[\delta_{ag} KT_{ag}^{-p} + (1 - \delta_{ag}) LD_{ag}^{-p} \right] \left(\frac{1}{p_{ag}} \right) \quad (9)$$

Composite labour demand derived from 9

$$LD_{ag} = \left(\frac{VA_{ag}}{B_{ag}} \right) \left[(1 - \delta_{ag}) + \delta_{ag} \left[\frac{\delta_{ag} w_{ag}}{(1 - \delta_{ag}) P K_{ag}} \right]^{\sigma_{ag} - 1} \right]^{\frac{1}{\rho_{ag}}} \quad (10)$$

C-D between agricultural capital and land

$$KT_{ag} = A_{ag} K_{ag}^{\alpha_{ag}} LAND_{ag}^{1 - \alpha_{ag}} \quad (11)$$

Agricultural capital derived from 11.

$$K_{ag} = \frac{\alpha_{ag} P K_{ag} KT_{ag}}{ra} \quad (12)$$

CES between skilled and unskilled labour labofooter

$$LD_{ag} = B_{ag}^l \left[\delta_{ag}^l LNQ_{ag}^{l - \rho_{ag}} + (1 - \delta_{ag}^l) LQ_{ag}^{l - \rho_{ag}} \right]^{\frac{1}{\rho_{ag}}} \quad (13)$$

$$LQ_{ag} = \left(\frac{LD_{ag}}{B_{ag}^l} \right) \left[(1 - \delta_{ag}^l) + \delta_{ag}^l \left[\frac{\delta_{ag}^l w_{ag}}{(1 - \delta_{ag}^l) w n_{ag}} \right]^{\sigma_{ag}^l - 1} \right]^{\frac{1}{\rho_{ag}}} \quad (14)$$

Factor prices

$$Pk_{ag} = \frac{Pva_{ag}VA_{ag} - w_{ag}LD_{ag}}{KT_{ag}} \quad (15)$$

$$rt_{ag} = \frac{Pk_{ag}KT_{ag} - raKD_{ag}}{LAND_{ag}} \quad (16)$$

$$w_{ag} = \frac{wnqLNQ_{ag} + wqLQ_{ag}}{LD_{ag}} \quad (17)$$

Income and savings

$$YH_h - wnq\lambda_h^{nq} \sum_i LNQ_i + wq\lambda_h^q \sum_i LQ_i + \lambda_h^k \left(\sum_{in} r_{in} KD_{in} + ra \sum_{ag} KD_{ag} \right) \quad (18)$$

$$+ \lambda_h^t \sum_{ag} rt_{ag} LAND_{ag} + TGH_h$$

$$YDH_h = YH_h(1 - ty_h) \quad (19)$$

$$YF = \left(1 - \sum_h \lambda_h^k \right) \left(\sum_{in} r_{in} KD_{in} + ra \sum_{ag} KD_{ag} \right) \quad (20)$$

$$YG = \sum_h ty_{hh} YH_h + \sum_i TXS_i + \sum_{com} TXM_{com} \quad (21)$$

$$TXS_i - tx_i P_i XS_i \quad (22)$$

$$TXM_{com} = \alpha m_{com} \overline{PmW}_{com} M_{com} \quad (23)$$

$$SH_h = mps_h YDH_h \quad (24)$$

$$SF = YF \quad (25)$$

$$SG = YG - \sum_h TGH_h - CG \quad (26)$$

Demand for commodities

$$CH_h - YDH_h - SH_h \quad (27)$$

$$C_{hcom} = \frac{Pq_{com}\bar{\omega}_{h,com} + \beta_{h,com}^c \left(CH_h - \sum_{com} \bar{\omega}_{h,com} Pq_{com} \right)}{Pq_{com}} \quad (28)$$

$$CT_{com} = \sum_h C_{h,com} + \frac{\beta_{com}^g \overline{CG}}{Pq_{com}} \quad (29)$$

$$INTDcom = \sum_j ICJcom, j \quad (30)$$

$$INVcom = \frac{\beta_{com}^i IT}{Pqcom} \quad (31)$$

Prices

$$Pvai = \frac{PiXSi - \sum_{com} Pqcom ICJcom, i}{VAi} \quad (32)$$

$$Pm_{com} = \overline{Pwm_{com}} (1 + tm_{com}) e^{-r} \quad (33)$$

$$Pe_{mer} = \frac{\overline{Pwe}_{mer} \bar{e}}{(1 + te_{mer})} \quad (34)$$

$$pq_{com} = \frac{PdcomDcom + PmcomMcom}{Qcom} \quad (35)$$

$$Pdser = \frac{PdservDserv - PdasDas}{Dser} \quad (36)$$

$$Pd_{as} = (1 + tx_{as}) P_{as} \quad (37)$$

$$P_{mer} = \frac{Pd_{mer} D_{mer} + Pe_{mer} EX_{mer}}{(1 + tx_{mer}) XS_{mer}} \quad (38)$$

$$Pindex = \sum_i \beta_i^x P_i \quad (39)$$

Foreign trade

$$XSas = Das \quad (40)$$

$$Dserv = A^d D_{as}^a \quad d D_{ser}^{1-\alpha^a} \quad (41)$$

$$Das = \frac{\alpha^d PDservDserv}{Pdas} \quad (42)$$

$$XS_{mer} = B_{mer}^T \left[\delta_{mer}^T EX_{mer} + \left(1 - \delta_{mer}^T \right) DP_{mer} \right]^{T \frac{1}{P_{mer}}} \quad (43)$$

$$EX_{mer} = \left(\frac{pe_{mer}}{Pd_{mer}} \right)^{\sigma_{mer}^t} \begin{pmatrix} (1 - \delta_{mer}^T) \\ \delta_{mer}^T \end{pmatrix}^{\sigma_{mer}^T} D_{mer} \quad (44)$$

$$Q_{com} = B_{com}^s \left[\delta_{com}^s M_{com} + \left(1 - \delta_{com}^s \right) D_{com} \right]^{s \frac{1}{p_{com}}} \quad (45)$$

$$M_{com} = \left(\frac{Pd_{com}}{Pm_{com}} \right)^{\sigma_{com}^s} \begin{pmatrix} \delta_{com}^s \\ (1 - \delta_{com}^s) \end{pmatrix}^{\sigma_{com}^s} D_{com} \quad (46)$$

$$CAB = \sum_{com} PWM_{com} M_{com} - \sum_{mer} Pwemer EX_{mer} \quad (47)$$

Equilibrium conditions

$$Q_{com} = CT_{com} + INTD_{com} + INV_{com} \quad (48)$$

$$LSNQ = \sum_i LNQ_i \quad (49)$$

$$LSQ = \sum_i LQ_i \quad (50)$$

$$KA = \sum_{ag} KD_{ag} \quad (51)$$

$$IT = SF + \sum_i SH_h + SG + e\overline{CAB} \quad (52)$$

Number of independent equations 223